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The Effect of Nonzero Autocorrelation Coefficients on the Distributions of Durbin-Watson Test Estimator: Three Autoregressive Models

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This paper investigates the effect of the nonzero autocorrelation coefficients on the sampling distributions of the Durbin-Watson test estimator in three time-series models that have different variance-covariance matrix assumption, separately. We show that the expected values and variances of the Durbin-Watson test estimator are slightly different, but the skewed and kurtosis coefficients are considerably different among three models. The shapes of four coefficients are similar between the Durbin-Watson model and our benchmark model, but are not the same with the autoregressive model cut by one-lagged period. Second, the large sample case shows that the three models have the same expected values, however, the autoregressive model cut by one-lagged period explores different shapes of variance, skewed and kurtosis coefficients from the other two models. This implies that the large samples lead to the same expected values, 2(1 – ρ_0*), whatever the variance-covariance matrix of the errors is assumed. Finally, comparing with the two sample cases, the shape of each coefficient is almost the same, moreover, the autocorrelation coefficients are negatively related with expected values, are inverted-U related with variances, are cubic related with skewed coefficients, and are U related with kurtosis coefficients.*

Keywords*: Nonzero autocorrelation coefficient, the d statistic, serial correlation, autoregressive model, time series analysis*

JEL Classification*: C32, C15, C52*

1. Introduction

Serial correlation has the most important role in autoregressive models, which is based on the regression analysis. If the data has serial correlation, then the researchers have to pay attention to it and use the correct variance-covariance matrix for estimation and forecasting. However, Lee (2014a) indicates the reasons of the difference between the errors and the residuals in regression analysis where $\mathbf{X}^T\hat{\boldsymbol{\varepsilon}} = \mathbf{0}$ is its internal constraint for the residuals, that is also affected by the values of the independent variables. Another one important factor is degree of freedom which constraints the relationship of sample size and the number of independent variables. Lee found that $\mathbf{X}^T\hat{\boldsymbol{\varepsilon}} = \mathbf{0}$ is very important when the degree of freedom is not very large in the regression analysis, thus, the autoregressive model will have to pay considerable attention on the above factors when the researchers use the serial correlation test estimator. Lee (2014b) also discusses the Z

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test that can be used in the serial correlation test of the *d* statistic when the degree of freedom is larger than 50. In that paper, Lee investigated the effects of the factors, including the variances of the errors, the values of the independent variables, on the distributions of the *d* statistic. Therefore, we do not repeat the investigations in this paper.

Due to the internal constraint and degree of freedom, we use three models, including Durbin-Watson model (Durbin and Watson, 1950, 1951), the common autoregressive model, and autoregressive model with one-lagged period, AR(1) model, (Savin and White, 1978), to discuss the effect of nonzero autocorrelation coefficients on the distributions of the Durbin-Watson test estimator, the *d* statistic. The reason we choose the *d* statistic is that its formula is the combination of the residuals and no one researches from the viewpoint of the degrees of freedom. In fact, the Durbin-Watson model has unfixed variance-covariance matrix, and the AR(1) model is restricted in the range from -0.5 to 0.5. We intend to show the differences from the distributions of the *d* statistic among three models, and to compare the coefficients of the *d* statistic between any two models.

This paper complements and explains if the null hypothesis is the nonzero autocorrelation coefficients, H_0: $\rho = \rho_0$, $\rho_0 \neq 0$, then how the distributions of the *d* statistic will become and what are the differences among three models. We show that the three models have the same autocorrelation coefficients as the null hypothesis in the robust analysis, but in the small samples, the three models have different distributions of the *d* statistic. It is worthy noting that the importance of the null hypothesis with the nonzero autocorrelation coefficient. When the researchers can know the data of the exactly autocorrelation coefficient, they can accurately forecast and judge the future. Even the critical value table can be built without neglecting the properties of the errors and the values of the independent variables. The structure of the paper is as follows. Section 2 describes the three model settings and the simulation procedure. Section 3 explores our simulation results that have (1) the patterns of four coefficients among three models when the sample is 57 and the number of independent variables is 6, and (2) the patterns of four coefficients between any two models when the sample is 1000 and the number of independent variables is 6. Section 4 presents the conclusions and discussion of the results.

2. The model

Consider a linear regression model with k regressors and T sample sizes, as

$$\underset{(T\times 1)}{\mathbf{Y}} = \underset{(T\times k)}{\mathbf{X}}\ \underset{(k\times 1)}{\mathbf{B}} + \underset{(T\times 1)}{\mathbf{e}}$$

Each ε_t is the error matrix, **e**, and satisfied with three conditions that are

(i) ε_t is i.i.d. Normal distribution.
(ii) $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma^2$ for all t.
(iii) $E(\varepsilon_t \times \varepsilon_{t-1}) = 0$ and $E(\varepsilon_i \times \varepsilon_j) = 0$, $|i - j| > 1$, for all t and i, j = 1, 2, …, T.

$\mathbf{Y} = \mathbf{X}\ \mathbf{B} + \mathbf{e}$ is constrained by $E(\mathbf{e}) = 0$ and $E(\mathbf{X}^T\mathbf{e}) = 0$. Use OLS and get the estimator of coefficients, $\hat{\mathbf{B}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$, due to the constraint of $\mathbf{X}^T\mathbf{e} = 0$. Thus the residuals are $\hat{\boldsymbol{\varepsilon}} = (\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T)\boldsymbol{\varepsilon}$, which is satisfied with $E(\hat{\boldsymbol{\varepsilon}}) = 0$ and $\mathbf{X}^T\hat{\boldsymbol{\varepsilon}} = 0$ and the degree of freedom being T-p-1. The sum of square residuals will be

$$E(\hat{\boldsymbol{\varepsilon}}\hat{\boldsymbol{\varepsilon}}^T) = \sigma^2(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T) \quad (1)$$

The condition (iii) guarantees the errors are independent from each other. However, the serial correlation model has broken condition (iii). In order to test the extensibility of the *d* statistic in the serial correlation models, the models are that

- Model A is the Durbin-Watson model introduced by Durbin and Watson in 1950.
- Model B is the serial correlation model in common. This is also the benchmark model.
- Model C is the autoregressive error procedure with one lagged period.

2.1. Model A

The serial correlative condition of Model A is $\varepsilon_{t+1} = \rho\varepsilon_t + \mu_{t+1}$, where t = 0, 1, …, T-1, ρ is population autocorrelation coefficient of ε_{t+1} and ε_t, μ_{t+1} is i.i.d. Normal distribution with $E(\mu_{t+1}) = 0$ and $Var(\mu_{t+1}) = \sigma^2$ for all t. The specialist property of Model A is the unfixed variance of the error when t increases, that is,

$$\varepsilon_{t+1} = \rho\times\varepsilon_t + \mu_{t+1} = \sum_{j=1}^{t+1}\left(\rho^{t+1-j}\times\mu_j\right),$$

$$Var(\varepsilon_{t+1}) = \sigma^2\sum_{j=1}^{t+1}\left(\left(\rho^2\right)^{t+1-j}\right)$$

$$E(\varepsilon_t\times\varepsilon_{t+1}) = \rho\times\left(\sum_{j=1}^{t}\left(\left(\rho^2\right)^{t+1-j}\right)\right)\times\sigma^2,$$

$$\rho(\varepsilon_t,\varepsilon_{t+1}) = \frac{\rho\times\left(\sum_{j=1}^{t}\left(\left(\rho^2\right)^{t+1-j}\right)\right)}{\sqrt{\sum_{j=1}^{t}\left(\left(\rho^2\right)^{t+1-j}\times\mu_j\right)\times\left(\sum_{j=1}^{t+1}\left(\left(\rho^2\right)^{t+1-j}\times\mu_j\right)\right)}},$$

where $\rho(\varepsilon_t, \varepsilon_{t+1})$ is the sample autocorrelation coefficient. If t approaches to infinite, then $Var(\varepsilon_t) = \sigma^2 / (1 - \rho^2)$, $E(\varepsilon_t \times \varepsilon_{t+1}) = \rho\, Var(\varepsilon_t)$ and $\rho(\varepsilon_t, \varepsilon_{t+1}) = \rho$ (see the proofs in Appendix I).

2.2. Model B

The serial correlation condition in Model B is $\varepsilon_{t+1} = \rho\varepsilon_t + \mu_{t+1}$, where $E(\varepsilon_t \times \varepsilon_{t+1}) = \rho\sigma^2$, t = 1, 2, …, T-1, thus, μ_{t+1} is *i.i.d.* Normal distribution with $E(\mu_{t+1}) = 0$ and $Var(\mu_{t+1}) = (1 - \rho^2)\,\sigma^2$ for all t. This serial correlation condition indicates the conditional ε_{t+1} on ε_t is Normal distribution with $E(\varepsilon_{t+1} \mid \varepsilon_t) = \rho\varepsilon_t$ and $Var(\varepsilon_{t+1} \mid \varepsilon_t) = (1 - \rho^2)\,\sigma^2$. Therefore, the variance-covariance matrix is

$$E(\varepsilon\varepsilon^T) = \sigma^2\begin{bmatrix} 1 & \rho & \rho^2 & \ldots & \rho^{T-2} & \rho^{T-1} \\ \rho & 1 & \rho & \ldots & \rho^{T-3} & \rho^{T-2} \\ \rho^2 & \rho & 1 & \ldots & \rho^{T-4} & \rho^{T-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho^{T-2} & \rho^{T-3} & \rho^{T-2} & \ldots & 1 & \rho \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \ldots & \rho & 1 \end{bmatrix},$$

The special property of Model B is

$$E(\varepsilon_t\times\varepsilon_{t+1}) = \rho\times\sigma^2,$$

$$\rho(\varepsilon_t,\varepsilon_{t+1}) = \frac{\rho\times\sigma^2}{\sqrt{\sigma^2\times\sigma^2}} = \rho\ .$$

2.3. Model C

The serial correlation condition in Model C is $\varepsilon_{t+1} = \rho\varepsilon_t + \mu_{t+1}$, where $E(\varepsilon_t \times \varepsilon_{t+1}) = \rho\sigma^2$, t = 1, 2, …, T-1, and $E(\varepsilon_t \times \varepsilon_{t+j}) = 0$, j > 1. Thus, the variance-covariance matrix is

$$E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{T}) = \begin{bmatrix} \sigma^2 & \rho\times\sigma^2 & 0 & ..... & 0 & 0 \\ \rho\times\sigma^2 & \sigma^2 & \rho\times\sigma^2 & ..... & 0 & 0 \\ 0 & \rho\times\sigma^2 & \sigma^2 & ..... & 0 & 0 \\ ..... & ..... & ..... & ..... & ..... & ..... \\ 0 & 0 & 0 & \rho\times\sigma^2 & \sigma^2 & \rho\times\sigma^2 \\ 0 & 0 & 0 & ..... & \rho\times\sigma^2 & \sigma^2 \end{bmatrix}$$

It should be noted that the autocorrelation coefficients cannot be more than 0.5 and less than -0.5, or the model would be flawed.

2.4. The Durbin-Watson test

As to ρ = 0, the three models become one model where the errors *i.i.d.* Normal distribution with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma^2$. Thus, the joint probability density function of the errors is

$$f\left(\boldsymbol{\varepsilon}_1,\cdots,\boldsymbol{\varepsilon}_T\right) = \left(\sigma\sqrt{2\pi}\right)^{-T} e^{-\left(\sum_{t=1}^{T}\varepsilon_t^2\right)\left(2\sigma^2\right)^{-1}}$$

where $-\infty < \varepsilon_t < \infty$ and t = 1, 2,…,T, and then those residuals that are calculated from the Original Least Square (OLS) method will be also restricted by the internal constraint, $\mathbf{X}^{T}\hat{\boldsymbol{\varepsilon}} = 0$. The DW test statistic is not noised by σ^2. Unfortunately, the lack of discussions of the *d* statistic is not only property of central limited theorem, that has been discussed by Lee (2014b), but also the effect of nonzero autocorrelation coefficients on the distributions of the *d* statistic which has different variance-covariance matrices in three models. As to the hypotheses, H_0: $\rho = \rho_0$ and H_1: $\rho \neq \rho_0$, the joint probability density function is

$$f\left(\varepsilon_1,\dots,\varepsilon_T\right) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^{T} \times e^{-\frac{\boldsymbol{\varepsilon}^T\boldsymbol{\Sigma}\boldsymbol{\varepsilon}}{2|\boldsymbol{\Sigma}|^{0.5}}},$$

where $\Sigma = E(\hat{\boldsymbol{\varepsilon}}\hat{\boldsymbol{\varepsilon}}^{T})$ and $-\infty < \varepsilon_t < \infty$ and t = 1, 2,…, T.

Durbin and Watson (1950, 1951) build the *d* statistic for testing the serial correlation of the data when the null hypothesis is a zero autocorrelation coefficient. The *d* statistic is

$$DW = \frac{\sum_{t=1}^{T-1}\left(\hat{\varepsilon}_t - \hat{\varepsilon}_{t+1}\right)^2}{\sum_{t=1}^{T}\hat{\varepsilon}_t^2}$$

where $\hat{\varepsilon}_t = Y_t - \hat{Y}_t$. However, the mathematical transformations of joint probability density functions, from the errors to the residuals and from the residuals to the *d* statistic, are not 1-1 relation and cannot find the jocabian functions, that is,

$$f\left(\hat{\varepsilon}_1,\dots,\hat{\varepsilon}_{T-1}\right) = f\left(\varepsilon_1,\dots,\varepsilon_T\right) \times \frac{\partial\left(\varepsilon_1,\dots,\varepsilon_T\right)}{\partial\left(\hat{\varepsilon}_1,\dots,\hat{\varepsilon}_{T-1}\right)},$$

and

$$DW = f\left(\hat{\varepsilon}_1,\dots,\hat{\varepsilon}_{T-1}\right) \times \frac{\partial\left(\hat{\varepsilon}_1,\dots,\hat{\varepsilon}_{T-1}\right)}{\partial\left(DW\right)}.$$

Because the exactly sampling distributions of the *d* statistic cannot be found, the computer simulation is needed whatever the autocorrelation coefficient is zero or nonzero.

3. Simulation procedure

The computations are performed in C++ on Intel Core i7 desktop. In order to control the internal constraint of regression and to derive the probability density functions of the *d* statistic, we use a new simulation process based on random number method to overcome the problems of probability transformation.† Thus, the Durbin-Watson test estimator can be simulated under null hypothesis, H_0: $\rho = \rho_0$, where $\rho_0 \neq 0$. The computer repeats 2^{16} calculations per time to get 2^{16} values of the Durbin-Watson test. The research method is as followed.

Step 1: Give the intercept and slope value, $\beta_0 = \beta_1 = \beta_2 = \ldots = \beta_k = 0$, and the data set of independent variables.

Step 2: Get the error data set of normal distribution which sample size is *T*. Here, the error value is independently.

Step 3: According to the linear regression model setting and computing the data set of dependent variable, $\mathbf{Y} = \mathbf{XB} + \boldsymbol{\varepsilon}$.

Step 4: Calculate the point-estimated values of regression coefficient and getting the estimated values of dependent variable, $\hat{\mathbf{Y}} = \mathbf{X}\hat{\mathbf{B}}$.

Step 5: Calculate the data set of residual, $\hat{\boldsymbol{\varepsilon}} = \hat{\mathbf{Y}} - \mathbf{X}\hat{\mathbf{B}}$.

Step 6: Get the value of the *d* statistic.

Every time generate 2^{16} values by repeating Step 2 to Step 6. Those values can generate a frequency table and then calculate the sampling distributions and coefficients. Because 2^{16} values per time is large enough, the sampling distributions of the *d* statistic can be viewed as population distributions. The error of coefficients between real value and estimated value is from 1/1000 to 1/10000.

When the sampling distributions of the *d* statistic are generated, the computer calculates the means, variances, skewedness, and kurtosis coefficients. The skewedness and kurtosis coefficients can ensure more whether the sampling distributions of the *d* statistic are Normal distribution or not in the three models. The paper defines the coefficients of the *d* statistic as

- ρ = X1 is the autocorrelation coefficient of the errors.
- E(DW) = X2 is the mean of the *d* statistic.
- Var(DW) = X3 is the variance of the *d* statistic.
- σ(DW) = X4 is the standard deviation of the *d* statistic.
- γ_1(DW) = X5 is the skewedness of the *d* statistic.
- γ_2(DW) = X6 is the kurtosis of the *d* statistic.

4. Simulation results

First, the computer calculation depends on the values of independent variables (Appendix II), 6 regressors, the variance and autocorrelation coefficients of the errors. The sampling distributions of the *d* statistic have four coefficients which are patterned by the autocorrelation coefficients of the errors from -0.99 to 0.99 for Model A and Model B, and form -0.49 to 0.49 for Model C, as shown in Table 1. The small sample case, T = 57, shows the effect of the autocorrelation coefficient on the coefficients of the sampling distributions of the *d* statistic.

Table 1. *The extreme values of the coefficients of in three models when the autocorrelation coefficient is nonzero (T = 57, k =6)*

		X2	X3	X4	X5	X6
Model A	Max	3.659836288	0.066942063	0.258731643	0.775942865	5.882692292
	Min	0.496834365	0.020959918	0.144775404	-1.447411589	2.897774026

† The software of Durbin-Watson test is provided by C.C.C. Ltd. The software of Durbin-Watson test model (Model B) is available online on the website: https://www.facebook.com/pages/Welsh-Corgi-Program/606775822740593. The traditional Durbin-Watson test model is based on Imhof (1960) and Pan (1968).

Model B	Max	3.680587671	0.066940095	0.258727839	0.806132468	7.191216908
	Min	0.492612374	0.01518781	0.123238833	-1.736253752	2.896743261
Model C	Max	2.807037254	0.066956001	0.258758576	0.230931679	3.039399415
	Min	1.168332061	0.039141875	0.197843058	-0.237106198	2.898300265

Table 1 illustrates the maximum and minimum of five coefficients in three models. By comparison of Model A and B, two models have the same minimum of E(DW), maximum of Var(DW) and σ(DW), but slightly different maximum of E(DW), minimum of Var(DW) and σ(DW). Moreover, Model B is more positive-skewed and centralized than Model A. However, Model C has the most extreme differences of E(DW), γ_1(DW) and γ_2(DW) than Model A and Model B, except for maximum of Var(DW) and σ(DW).

Although Table 1 shows the five coefficients of the *d* statistic, we still do not know the effect of autocorrelation coefficients on the sampling distribution of the *d* statistic. Therefore, Table 2 illustrates the plots of four coefficients where the vertical axis is E(DW), Var(DW), γ_1(DW) and γ_2(DW), separately, and the horizontal axis represents the autocorrelation coefficients. Those plots assist us to investigate whether the *d* statistic is Normal distribution and how the autocorrelation coefficients affect the sampling distribution of the *d* statistic. As to the whole range of ρ in three models, E(DW) is negatively and linearly related with ρ. This implies that dE(DW) / $d\rho < 0$. The plot of E(DW) also passes through around 2.0028 (Model A and B) and 2.0027 (Model C) as $\rho = 0$. The reason is that the negative ρ causes the errors and the residuals to fluctuate up and down from t to t + 1 period, however, the positive ρ leads to one and fixed direction for the errors and residuals. Second, Var(DW) is an inverted-U shape. This implies that the higher the |ρ| is, the lower the Var(DW) is. However, the maximum of Var(DW) occurs at $\rho = 0.02$ in Model A, at $\rho = 0.01$ in Model B and at $\rho = -0.01$ in Model C. This also shows that the maximum of Var(DW) is not at $\rho = 0$. This is because the different assumption of variance-covariance matrix. From the view of E(DW) and Var(DW), E(DW) cannot be used to derive Var(DW) because the linear relationship cannot represent the U-shape relationship, especially when the autocorrelation coefficient is nonzero.

Table 2 also illustrates the plots of the skewed and kurtosis coefficients. The plots of skewed coefficient are cubic shape which shows the higher the ρ is, the higher the skewed coefficient is. The skewed coefficients are positive when $\rho > 0$. Model C has considerable shape of skewed coefficients by comparison with Model A and Model B. Although the kurtosis coefficient is 2.8978 in Model A, 2.8975 in Model B, and 208990 in Model C when $\rho = 0$, the minimum of the kurtosis coefficient occurs in $\rho = 0.01$ in three models. If |ρ| becomes larger than 0.01, the kurtosis coefficient increases, in particular, the higher the negative ρ is, the higher the kurtosis coefficient is in Model A and Model B. The plots show that the assumptions of Model B lead to the higher kurtosis coefficient than Model A when |ρ| becomes larger and close to higher relation. We also find that the kurtosis coefficient is larger than 3 when $\rho \leq -0.37$ in three models, but occurs when $\rho > 0.39$ in Model B and Model C, and when $\rho > 0.38$ in Model A.

Thus, we can obtain the proposition as follows.

Proposition 1.

(1) dE(DW) / $d\rho < 0$. When $\rho = 0$, E(DW) = 2.00 accurate to the second decimal place.

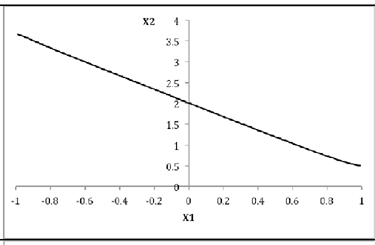
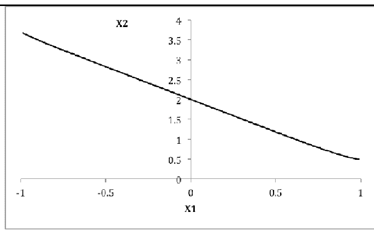
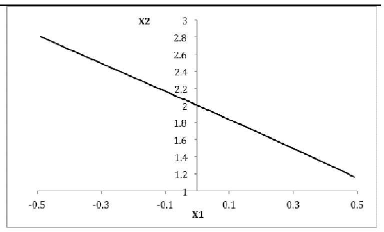
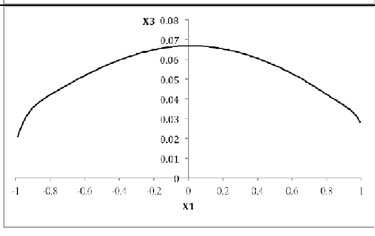
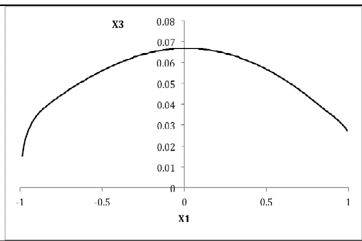
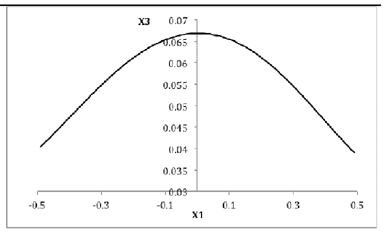
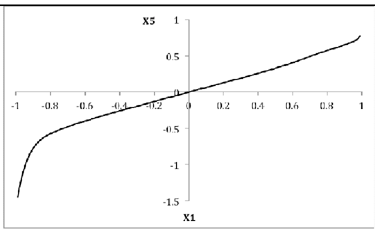
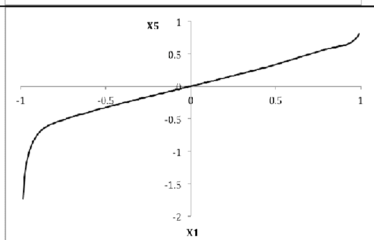
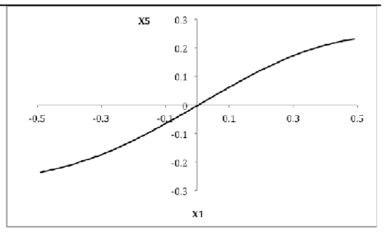
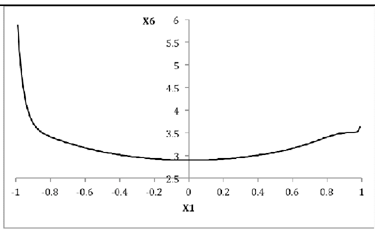
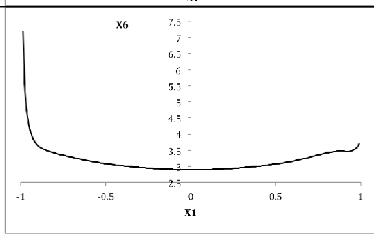
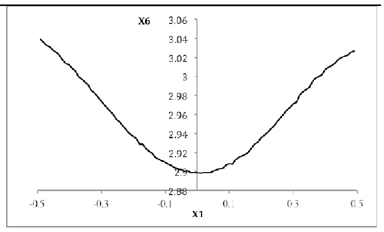
(2) When $\rho \leq 0$, dVar(DW) / $d\rho > 0$ and dVar(DW) / $d\rho < 0$ when $\rho > 0$. The second-order condition is d^2Var(DW) / $d\rho^2 < 0$.

(3) $d\gamma_1$(DW) / $d\rho > 0$. When $\rho = 0$, γ_1(DW) = -0.00 accurate to the second decimal place.

(4) When $\rho \leq 0$, $d\gamma_2$(DW) / $d\rho < 0$ and $d\gamma_2$(DW) / $d\rho > 0$ when $\rho > 0$. The second-order condition is $d^2\gamma_2$(DW) / $d\rho^2 > 0$.

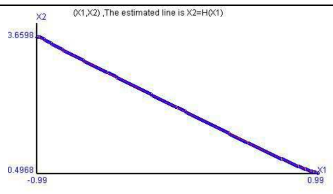
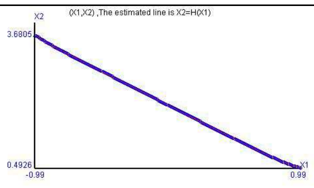
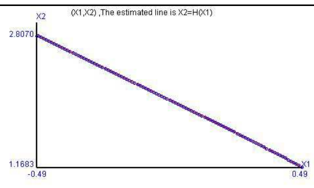
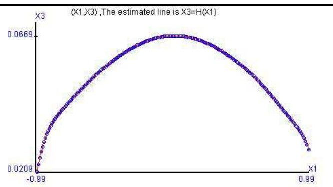
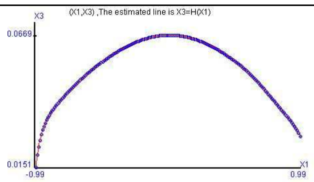
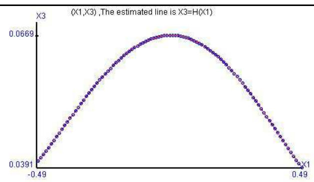
In the small sample case, the assumptions of $E(\varepsilon_t \times \varepsilon_{t+1})$ and the variance- covariance matrix leads to the differences among three models when the samples and the number of regressors are the same. This is because Model B has the fixed variance in the assumption of first-order autoregressive errors, meanwhile, Model C is based on the only one-lagged period effect on the errors. The four coefficients in three models show that the sampling distributions of the *d* statistic are not Normal distribution in the range of ρ. One reason is from Lee (2014a, 2014b), other reason is that nonzero ρ disturbs the errors, the residuals, its mathematical combination and the variance-covariance matrix, thus, the *d* statistic cannot display a Normal distribution.

Table 2. *The coefficients in three models when the autocorrelation coefficient is nonzero (T = 57, k =6)*

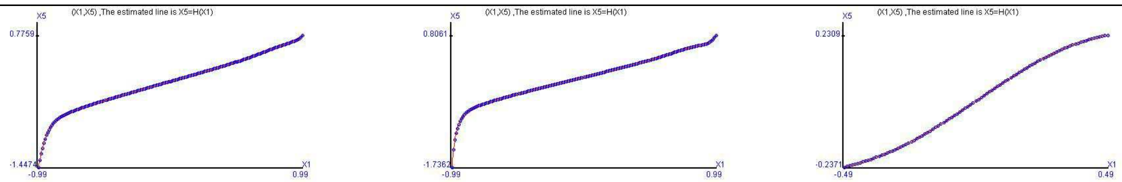
	Model A	**Model B**	**Model C**
X2			
X3			
X5			
X6			

Due to the relationship of each coefficient and ρ in Table 2, we can estimate each coefficient of the *d* statistic by the autocorrelation coefficients, that is, regress each coefficient on ρ by curve-linear regression method, which is based on the Taylor expansion function, in Table 3. The horizontal axis is the values of ρ and the vertical axis is the values of each coefficient. It is noted that the three models indicate that the *d* statistic is asymmetric at 2 and has significant difference in γ_1(DW) and γ_2(DW), in particular, when the null hypothesis is H_0: $\rho = \rho_0$.

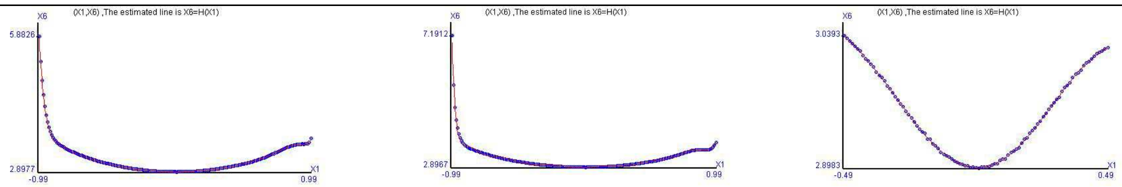
Table 3. *The estimation of each coefficient in three models when the autocorrelation coefficient is nonzero*

Symbol	**Model A**	**Model B**	**Model C**
X2			
X3			

X5



X6



The Appendix III illustrates the residual plots of each coefficient after estimation. The residual plots show that the model setting leads to the different effect of ρ on the coefficients of the *d* statistic, even the shapes of each coefficient are as similar as possible. Moreover, Model A and Model B have similar coefficients, but the residual plots are considerably different with each coefficient. The special variance-covariance matrix assumption leads to the residual plot of Model C different from others. Due to the considerable paths of coefficients as a change of autocorrelation coefficients, the *d* statistic is still sensitive and is used for hypothesis testing in three models when the null hypothesis is H_0: $\rho = \rho_0$.

4.1. Robust analysis

When the samples are large enough, the three models have the same sampling autocorrelation coefficient, $\rho(\varepsilon_t, \varepsilon_{t+1}) = \rho$, however, have different values of $E(\varepsilon_t \times \varepsilon_{t-1})$, that is,

$$E(\varepsilon_t, \varepsilon_{t+1}) = \begin{cases} \dfrac{\rho\sigma^2}{1-\rho^2} & , Model\ A \\ \rho\sigma^2 & , Model\ B, C \end{cases} .$$

Thus, the zero autocorrelation coefficient leads to $\rho(\varepsilon_t, \varepsilon_{t+1}) = \rho(\varepsilon_t, \varepsilon_{t+1}) = 0$ in the three models. If the null hypothesis is nonzero autocorrelation coefficient, H_0: $\rho = \rho_0$, and T is infinite, then

$$E(DW) = 2\ (1 - \rho_0). \qquad (2)$$

The expected values of the *d* statistic is a constant value away from 2 means that k has no impact on the robust means of the *d* statistic whatever the autocorrelation coefficient is. Furthermore, E(DW) is negatively and linear related with ρ_0 as shown in (2). However, E(DW) insufficiently represents the information of the sampling distributions of the *d* statistic when T is large enough. The second to fourth rows of Table 4 illustrate the effect of the higher moments on the sampling distributions of the *d* statistic. The second row shows that Model A and Model B have the same shape of variance, but are different from Model C. moreover, Var(DW) is affected considerably by the positive autocorrelation coefficient in Model A and by the negative autocorrelation coefficient in Model B. We also find that the higher the |ρ| is, the larger the difference between Model A and Model C (Model B and Model C) is.

The third row illustrates that the higher positive ρ leads to that (1) the skewed coefficients of Model A are larger than that of Model B and (2) there is a larger difference from the γ_1(DW) of Model A (or Model B) minus the γ_1(DW) of Model C. However, the larger negative ρ induces in the larger difference from γ_1(DW) of Model C minus γ_1(DW) of Model A (or Model B). We also can find that

$$\frac{d\gamma_1(DW)}{d\rho} \geq 0 .$$

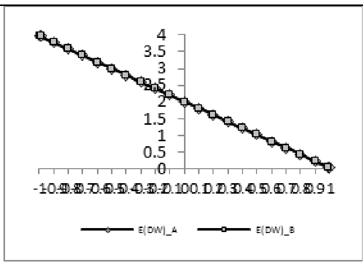
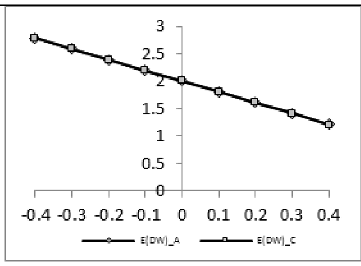
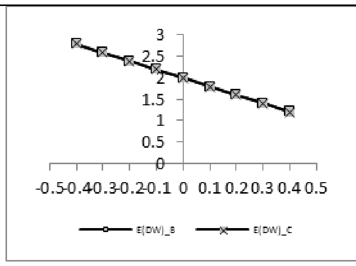
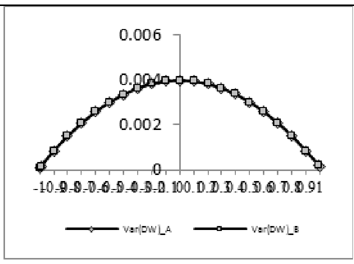
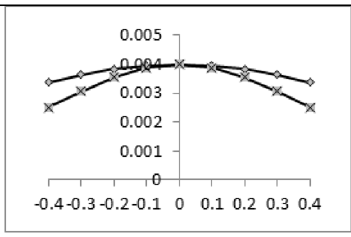
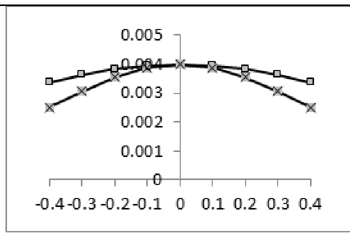
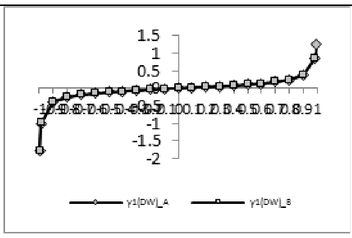
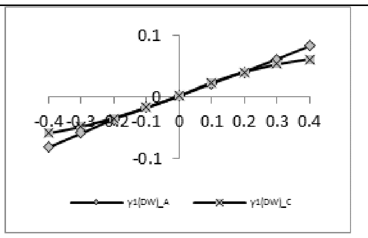
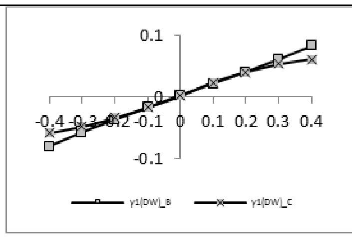
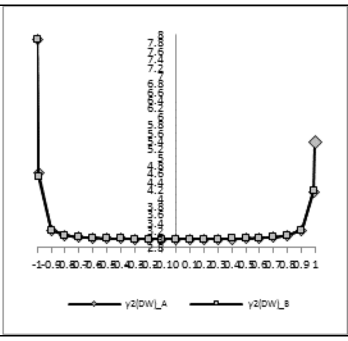
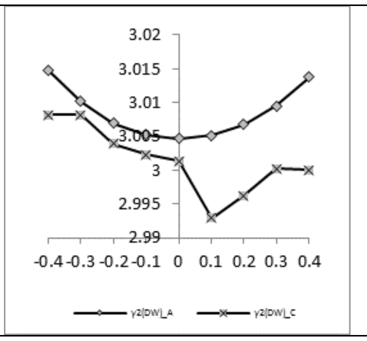
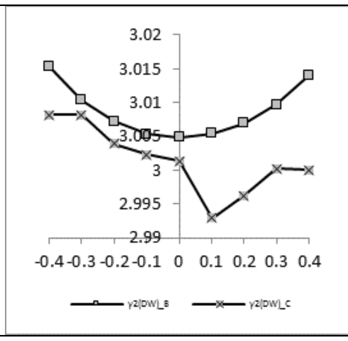
The fourth row illustrates that the smallest values of γ_2(DW) occurs in $\rho = 0$. When the $|\rho|$ becomes larger, γ_2(DW) increases in the three models, in particular, the pattern of γ_2(DW) has a kinked point at $\rho = 0.1$. The relationship between γ_2(DW) and ρ is

$$\frac{d\gamma_2(DW)}{d\rho}\begin{cases} >0 & ,if\ \rho<0 \\ <0 & ,if\ \rho>0 \end{cases},$$

and

$$\frac{d\gamma_2^2(DW)}{d\rho^2}<0.$$

***Table 4**. The comparison of the changes between three models' coefficients and autocorrelation coefficient (T =1000 and k =6)*

	Model A vs Model B	**Model A vs Model C**	**Model B vs Model C**
E(DW)			
Var(DW)			
skewness			
kurtosis			

Comparing with Table 2 and 4, the sampling distributions of the *d* statistic have the following properties.

Proposition 2. At small and large sample cases,
(1) E(DW) passes through 2 when null hypothesis is $\rho = 0$ in three models.

(2) E(DW) is negatively and linearly related with ρ in three models
(3) Var(DW) is inverted-U related with ρ in three models.
(4) Model A and B have the same shape of Var(DW), γ_1(DW) and γ_2(DW), that are different from Model C.

Table 4 also shows that the sampling distribution of the *d* statistic in three models is a Normal distribution with E(DW) = 2 due to γ_1(DW) = 0 and γ_2(DW) = 3. This is an evidence of Durbin and Watson (1950, 1951) when the samples are large enough. The second property is that the higher the positive ρ is, the more positive-skewed the sampling distributions of the *d* statistic are. The small sample case in Table 3 explains that the different variance-covariance matrix assumption affect the effect of the autocorrelation coefficient on the sampling distributions of the *d* statistic, that have different expected values and other coefficients as shown in Table 1, 2 and 3. When the samples become large, Table 4 shows the same expected value, variance and skewed coefficients among three models even through the kurtosis coefficient is a little bit different among the three models. Therefore, the large samples can eliminate the variance-covariance matrix assumption and lead the three models to become one model.

5. Conclusions

The paper runs computer simulation of serial correlation test for an example of Durbin-Watson test estimator when the errors have nonzero autocorrelation coefficient in first-order autoregressive model. We try to compare three models to show the effect of nonzero autocorrelation coefficients on the sampling distributions of the *d* statistic.

The results can be divided with three parts. The first result is from the viewpoint of the sample size. We find that whatever the sample size is, the expected values, variances, skewed and kurtosis coefficients have the same patterns of the autocorrelation coefficients in three models, separately, but part of values are not the same. We also find that the assumptions of variance-covariance matrix can be eliminated by the increasing samples, therefore, the sampling distributions of the *d* statistic have the same expected values in three models whatever the autocorrelation coefficients are. This result implies that in the long run, three time series models have the same expected values, $2(1 - \rho_0)$, that are different from the small sample case due to the expected values of Model C.

The second result is from the view of the null hypothesis with zero autocorrelation coefficient. We show that the higher the positive autocorrelation coefficient is, the lower expected values and variances of the *d* statistic are, but the higher the skewed and kurtosis coefficients are. There are reversed results in the situation of the negative autocorrelation coefficients. The third result is from the perspective of whole patterns of each coefficient. We shows that the autocorrelation coefficients are negatively and linearly related with expected values, inverted-U related with variances, cubic related with skewed coefficients and U-quadratic related with kurtosis coefficients when the autocorrelation coefficient is from the minimum to the maximum in three models whatever the samples are. The three results can supplement the literatures about the serial correlation test for an example of the sampling distributions of the *d* statistic.

6. References

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Appendix I

The model of Durbin and Watson (1950) is shown in section 2. Based on $\varepsilon_{t+1} = \rho \times \varepsilon_t + \mu_{t+1}$ and $\varepsilon_0 = 0$, the first error is $\varepsilon_1 = \mu_1$ and then substitute into $\varepsilon_{t+1} = \rho \times \varepsilon_t + \mu_{t+1}$ and obtain the second error,

$$\varepsilon_2 = \rho \times \varepsilon_1 + \mu_2, E(\varepsilon_2) = 0, Var(\varepsilon_2) = (1+\rho^2) \times \sigma^2.$$

Thus, $E(\varepsilon_1 \times \varepsilon_2) = \rho \times \sigma^2$ and the sampling autocorrelation coefficient of the first and second errors is

$$\rho(\varepsilon_1, \varepsilon_2) = \frac{\rho}{\sqrt{(1+\rho^2)}}.$$

Following the same calculated step, we can derive that

$$\varepsilon_3 = \rho \times \varepsilon_2 + \mu_3 = \rho^2 \times \mu_1 + \rho \times \mu_2 + \mu_3,$$
$$E(\varepsilon_3) = 0, Var(\varepsilon_2) = (1+\rho^2+\rho^4) \times \sigma^2, E(\varepsilon_2 \times \varepsilon_3) = \rho \times (1+\rho^2) \times \sigma^2,$$
$$\rho(\varepsilon_2, \varepsilon_3) = \frac{(1+\rho^2)}{\sqrt{(1+\rho^2) \times (1+\rho^2+\rho^4)}},$$
$$\varepsilon_4 = \rho \times \varepsilon_3 + \mu_4 = \rho^3 \times \mu_1 + \rho^2 \times \mu_2 + \rho \times \mu_3 + \mu_4,$$
$$E(\varepsilon_3) = 0, Var(\varepsilon_2) = (1+\rho^2+\rho^4+\rho^6) \times \sigma^2, E(\varepsilon_3 \times \varepsilon_4) = \rho \times (1+\rho^2+\rho^4) \times \sigma^2,$$
$$\rho(\varepsilon_2, \varepsilon_3) = \frac{(1+\rho^2+\rho^4)}{\sqrt{(1+\rho^2+\rho^4) \times (1+\rho^2+\rho^4+\rho^6)}},$$

Thus, the t+1th error is that

$$\varepsilon_{t+1} = \rho \times \varepsilon_t + \mu_{t+1} = \sum_{j=1}^{t+1} (\rho^{t+1-j} \times \mu_j), t = 0, 1, 2, ..., T-1,$$
$$E(\varepsilon_{t+1}) = 0, Var(\varepsilon_{t+1}) = \left(\sum_{j=1}^{t+1} \left((\rho^2)^{t+1-j} \right) \right) \times \sigma^2,$$
$$E(\varepsilon_t \times \varepsilon_{t+1}) = \rho \times \left(\sum_{j=1}^{t} \left((\rho^2)^{t+1-j} \right) \right) \times \sigma^2,$$
$$\rho(\varepsilon_t, \varepsilon_{t+1}) = \frac{\rho \times \left(\sum_{j=1}^{t} \left((\rho^2)^{t+1-j} \right) \right)}{\sqrt{\sum_{j=1}^{t} \left((\rho^2)^{t+1-j} \times \mu_j \right) \times \left(\sum_{j=1}^{t+1} \left((\rho^2)^{t+1-j} \times \mu_j \right) \right)}}$$

If t becomes infinite, then

$$Var(\varepsilon_t) = \frac{\sigma^2}{(1-\rho^2)},$$

$$E(\varepsilon_t \times \varepsilon_{t+1}) = \rho \times \frac{\sigma^2}{(1-\rho^2)},$$

$$\rho(\varepsilon_t, \varepsilon_{t+1}) = \rho$$

Appendix II

The values of independent variables are as follows.

	X1	X2	X3	X4	X5	X6
1 :	8.3942796673,	5.9743248114,	10.5433072070,	4.4919705977,	3.4547532369,	0.3660935162,
2 :	11.0926182469,	8.5865059254,	6.3276410268,	2.3013060649,	1.4194860328,	1.2979850825,
3 :	11.5956287654,	4.1842520567,	7.5754211833,	8.8748009076,	9.2966283694,	9.9319530606,
4 :	5.1294293068,	-2.0229243067,	-3.7205994925,	-1.9710922902,	-2.1071922389,	-3.9631068919,
5 :	10.0930459757,	2.9087056925,	-1.2463797401,	-1.6367202069,	-1.3437181198,	-2.4391750503,
6 :	10.7591584850,	6.2449255029,	1.9952881995,	-1.5751044802,	-2.6548128155,	-1.4225324227,
7 :	11.6389242060,	9.8559733690,	19.6769166275,	20.4092287951,	21.7139250496,	21.3941172645,
8 :	8.9339369321,	9.6674110044,	6.7274054640,	6.5328385454,	5.6853699385,	5.5179365070,
9 :	9.9655867410,	3.0860552350,	3.3136013263,	1.7862885234,	2.9068953205,	2.7189989340,
10 :	13.8544611999,	10.0577880915,	3.5409810319,	-1.4431102299,	-1.8569863686,	-4.1283708796,
11 :	8.7572543061,	1.5409028398,	3.3691779308,	3.6215888116,	4.1015402009,	1.6426980000,
12 :	11.4333746774,	8.2238421572,	3.8054267172,	7.5846859124,	7.0073612155,	6.0419247516,
13 :	7.7931279457,	9.7999622586,	13.7500226003,	18.0159218476,	18.5204442419,	19.6681100550,
14 :	10.2407183608,	12.0302431166,	21.5510469818,	24.0161513894,	23.9080548620,	22.8212635916,
15 :	14.3323237799,	9.4305408181,	10.6476934619,	10.1736825488,	11.1187329437,	11.5326224000,
16 :	9.2031646401,	10.3699880329,	13.4563031661,	17.1566859980,	17.1761252852,	18.7154800011,
17 :	9.6889534618,	18.4917329222,	20.2756938987,	21.1204395704,	19.5081788708,	19.1797324594,
18 :	11.7050955005,	9.2397083531,	15.1044690444,	14.1883861443,	12.3015307628,	15.4676571064,
19 :	9.1971260006,	9.7093882312,	8.0743237967,	10.1626439416,	9.9055454481,	7.9878593242,
20 :	11.6707749148,	10.4567343750,	5.4346885708,	-0.9926995759,	-0.2580358331,	-0.7226713447,
21 :	11.0084985964,	12.7098983514,	6.8381258406,	4.2922825990,	5.9724619693,	9.0484380444,
22 :	10.4393336651,	20.4063934575,	28.1712423189,	25.2106896922,	27.0691680197,	24.3148963415,
23 :	9.2754712315,	8.4469984069,	11.1884566405,	16.2975452059,	15.6140416028,	16.8164249462,
24 :	11.0956560003,	19.5079769882,	25.0650104995,	33.9560034171,	33.8813717485,	31.1591859967,
25 :	10.0791634122,	16.6823583643,	11.4975529785,	3.6802821135,	3.6446917006,	-1.5133543254,
26 :	8.3487204240,	7.3381429614,	6.1028497337,	10.1956370626,	7.8041414071,	6.1380528833,
27 :	6.8463525509,	8.3279194586,	13.8199774185,	12.8437508964,	10.8282376376,	8.8860080326,
28 :	10.2020988906,	27.1915803537,	27.5917133148,	29.9451507344,	28.9996646890,	29.0885852288,
29 :	11.1117713869,	11.2269060716,	3.5639405330,	-3.5779110472,	-2.6505471172,	-2.4333384008,
30 :	12.0664318754,	16.8006310885,	12.7379497272,	13.1094040242,	14.5127143598,	19.0237276954,
31 :	14.1141812980,	14.8284107479,	19.3868646899,	16.6036456738,	16.3620462244,	15.8977919007,
32 :	14.4138413029,	19.1844447867,	24.2918371048,	29.4721002588,	30.1116915480,	32.1152744433,
33 :	13.3645790169,	9.4110987477,	11.5351840149,	16.2980802028,	16.3960191979,	10.9799564480,
34 :	9.5655683954,	4.8458408500,	-0.5970340237,	-5.1658276064,	-3.2146578374,	-1.0457040390,
35 :	11.5832033625,	12.2463389796,	16.8335732809,	12.3957409976,	12.6831959365,	11.3378073349,
36 :	9.6500026201,	18.1501333618,	20.0768361023,	26.4200908540,	26.6705510566,	26.4265966911,
37 :	12.3330653338,	13.0854669917,	10.3700300441,	3.1991677444,	3.3954099265,	2.7978941630,
38 :	8.6508259080,	7.4336577229,	3.3471939688,	-5.6158892589,	-5.7818123740,	-9.0915191233,
39 :	4.5136304186,	12.2936868514,	11.2979665482,	-4.2015056613,	-4.3902967105,	0.3980143565,
40 :	9.8912177478,	8.4305240403,	5.5554726620,	2.9828278842,	2.9390925524,	2.0756327884,
41 :	10.5036288232,	7.2971005321,	6.5609778313,	4.1186668666,	6.6299078094,	5.0101628919,
42 :	11.5565815704,	8.4932003482,	13.4741026052,	12.2157823319,	12.5357982707,	13.2931521944,
43 :	9.0099119726,	8.7630513853,	8.6233918610,	21.5107027433,	21.2282336352,	21.0894539511,
44 :	7.5743427980,	14.2064111846,	9.6177663912,	14.9992214910,	14.1796342689,	13.9101853988,
45 :	11.6365114336,	6.9482916849,	9.2461095245,	10.7606715127,	13.7441068642,	12.3564493977,
46 :	11.1134974482,	13.0029282291,	17.7683519485,	10.2323919463,	9.4664227563,	10.6690435813,
47 :	10.3679908394,	7.5178716331,	6.9816044043,	5.0161344127,	4.0434035535,	6.9792018302,
48 :	7.9550202448,	7.9756557376,	16.6152281740,	12.9908386728,	12.8983318498,	13.2096870593,
49 :	9.8991655726,	10.0250753674,	1.0786738992,	10.1037433773,	10.7185910088,	12.5231682908,
50 :	13.3109816579,	13.0867379409,	16.2703335687,	6.9847587587,	7.2106934802,	6.7330443978,
51 :	10.6629372996,	20.0562925451,	18.1713161972,	25.4100266454,	26.7295197714,	30.9814884118,
52 :	9.8145247150,	6.1691472249,	10.8781016015,	15.9769454371,	15.0685888697,	13.9958726602,
53 :	9.4802865631,	12.5350409185,	13.4437574182,	12.5934461630,	14.2171197996,	15.6588514033,
54 :	10.6029033874,	15.0919249001,	17.3438740646,	19.9795003428,	20.9217770362,	24.7741874651,
55 :	11.7298865621,	10.1802191618,	10.5720358408,	13.6541262143,	13.0717268281,	13.3738621297,
56 :	11.6520524994,	13.4732923593,	15.1137069458,	19.6391088533,	19.8817273292,	22.6044071700,
57 :	8.7944443244,	7.7765307842,	13.3726243393,	0.4885322455,	1.5605716470,	1.3088040036,

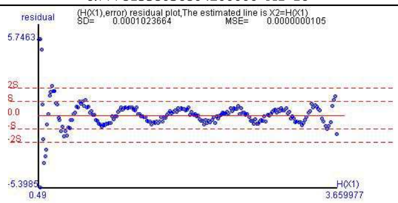
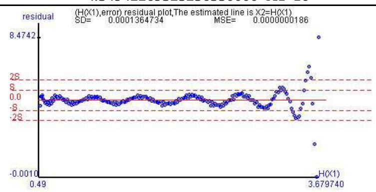
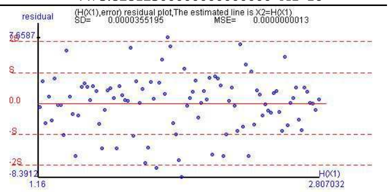
indepedent sample correlation coefficient------------

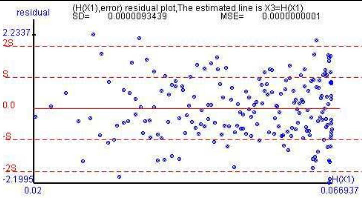
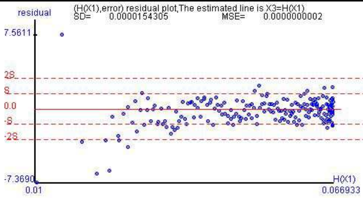
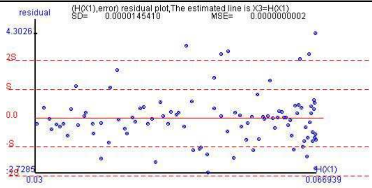
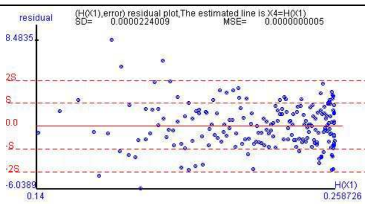
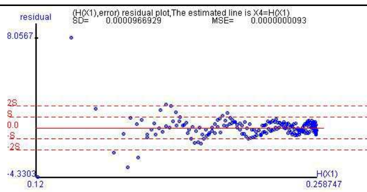
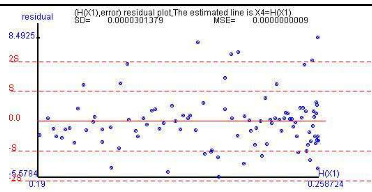
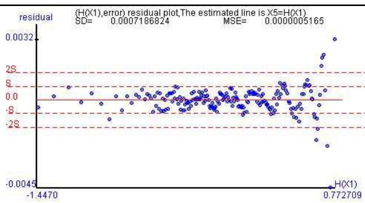
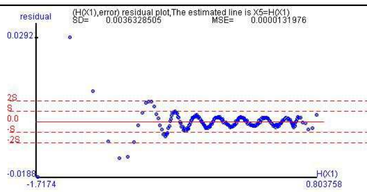
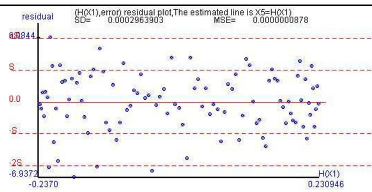
r(X1,X2)=	0.2859050169
r(X1,X3)=	0.2312889512
r(X1,X4)=	0.2005213829
r(X1,X5)=	0.2265887605
r(X1,X6)=	0.2167931260
r(X2,X3)=	0.7866728319
r(X2,X4)=	0.6583522828
r(X2,X5)=	0.6616447747
r(X2,X6)=	0.6699690814
r(X3,X4)=	0.8279580562
r(X3,X5)=	0.8262028589
r(X3,X6)=	0.8057327110
r(X4,X5)=	0.9937831984
r(X4,X6)=	0.9677570153
r(X5,X6)=	0.9770514117

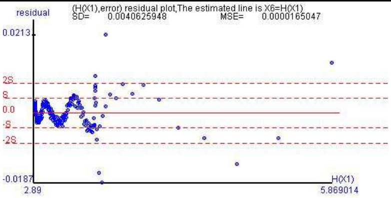
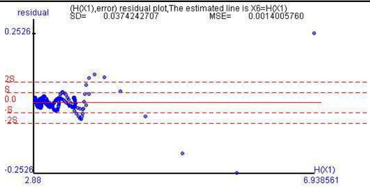
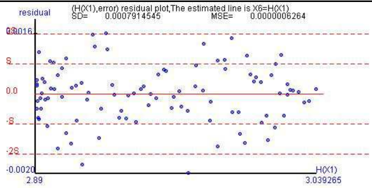
Appendix III

Table 3 shows the estimated line where we regress each coefficient of the *d* statistic on the autocorrelation coefficients in the autoregressive models with T = 57, k = 6 and three variance-covariance matrices. Appendix III shows the estimated function of each coefficient and the corresponding residual plot.

***Table A-III.** The estimated function and residual plot of each coefficient in three models*

	Model A	Model B	Model C
Estimated function of X2	X2= 2.00276415438656840000+ -1.64527441226274250000*X1^1+ 0.02402413310483098000*X1^2+ 0.04011142093775976700*X1^3+ -0.19569147005677223000*X1^4+ -0.07628797340541204600*X1^5+ 1.96844828128814700000*X1^6+ 0.47423921976266570000*X1^7+ -9.18280923366546630000*X1^8+ -1.92248008535207760000*X1^9+ 23.28459286689758300000*X1^10+ 4.14671876727263110000*X1^11+ -32.32774972915649400000*X1^12+ -4.30549033957667860000*X1^13+ 23.28432893753051800000*X1^14+ 1.69847563160714190000*X1^15+ -6.77781558036804200000*X1^16+	X2= 2.00285458297548760000+ -1.64470797005871040000*X1^1+ -0.00454932160209864380*X1^2+ 0.00927235203368426130*X1^3+ 0.16936979815363884000*X1^4+ 0.28804339144714852000*X1^5+ -1.49353519082069400000*X1^6+ -1.72851022418154090000*X1^7+ 6.88667500019073490000*X1^8+ 4.85288819303698920000*X1^9+ -16.79043436050415000000*X1^10+ -6.98888725740226850000*X1^11+ 22.59680891036987300000*X1^12+ 5.07867236837829240000*X1^13+ -15.62164354324340800000*X1^14+ -1.47609063876583240000*X1^15+ 4.34942269325256350000*X1^16+	X2= 2.00281731306768050000+ -1.64539365254915730000*X1^1+ -0.06201925780624151200*X1^2+ -0.09805036787599519200*X1^3+ 0.04113501310348510700*X1^4+ -0.18425352406529782000*X1^5+ 0.23702049255371094000*X1^6+ 0.77233835506740434000*X1^7+ -21.15847778320312500000*X1^8+ 3.66662154669757000000*X1^9+ 270.37036132812500000000*X1^10+ -39.18330250936560300000*X1^11+ -1543.53515625000000000000*X1^12+ 89.36592935863882300000*X1^13+ 4213.85546875000000000000*X1^14+ -28.53425410110503400000*X1^15+ -4475.82812500000000000000*X1^16+
X2			
Estimated function of X3	X3= 0.06693278714515330000+ 0.00096872929498204030*X1^1+ -0.04520157202942237700*X1^2+ -0.00275858333043288440*X1^3+ 0.02098693347819846600*X1^4+ 0.05889438558369875000*X1^5+ -0.04170386898277911300*X1^6+ -0.55974376387894154000*X1^7+ 0.10307001079297606000*X1^8+ 2.80556001514196400000*X1^9+ -0.19054156384027010000*X1^10+ -8.35619109869003300000*X1^11+ 0.10881430686413296000*X1^12+ 15.06040996313095100000*X1^13+ 0.19172376901815369000*X1^14+ -16.10446360707283000000*X1^15+ -0.27125440162717496000*X1^16+ 9.41034364700317380000*X1^17+ 0.07943948994756056000*X1^18+ -2.30899981409311290000*X1^19+	X3= 0.06692745159035373700+ 0.00100926802133471940*X1^1+ -0.04403440576186312700*X1^2+ -0.00818694464396685360*X1^3+ -0.02190339320600287500*X1^4+ 0.17682791128754616000*X1^5+ 0.56392564276029589000*X1^6+ -2.03245097398757930000*X1^7+ -4.36716125474777070000*X1^8+ 12.71202158927917500000*X1^9+ 18.62126043066382400000*X1^10+ -47.70089125633239700000*X1^11+ -47.93447231641039300000*X1^12+ 111.62678289413452000000*X1^13+ 75.95208441745489800000*X1^14+ -164.07429504394531000000*X1^15+ -72.36105220322497200000*X1^16+ 147.18862628936768000000*X1^17+ 38.04211827833205500000*X1^18+ -73.61198830604553200000*X1^19+ -8.50006912779645060000*X1^20+ 15.73104691505432100000*X1^21+	X3= 0.06693905433940017200+ 0.00112085084241542180*X1^1+ -0.14931126567535102000*X1^2+ -0.05720452920922980400*X1^3+ 0.23701512813568115000*X1^4+ 2.79059713566675780000*X1^5+ -5.54301834106445310000*X1^6+ -82.39533479511737800000*X1^7+ 162.23004150390625000000*X1^8+ 1337.14645475149150000000*X1^9+ -2696.40917968750000000000*X1^10+ -12839.57588338851900000000*X1^11+ 26713.79687500000000000000*X1^12+ 75017.66387939453100000000*X1^13+ -160488.68750000000000000000*X1^14+ -261974.91619873047000000000*X1^15+ 573819.75000000000000000000*X1^16+ 502634.39389038086000000000*X1^17+ -1124086.00000000000000000000*X1^18+ -407490.87664794922000000000*X1^19+ 929648.50000000000000000000*X1^20+

X3			
Estimated function of X4	X4= 0.25871634963780865000+ 0.00190481592380820080*X1^1+ -0.08788307533168926400*X1^2+ -0.00703720701858401300*X1^3+ 0.04243934395884707600*X1^4+ 0.16138683911412954000*X1^5+ -0.27570165432905469000*X1^6+ -1.52017983049154280000*X1^7+ 1.41675009857317490000*X1^8+ 7.59491324424743650000*X1^9+ -4.54018481299863200000*X1^10+ -22.45107614994049100000*X1^11+ 8.53803876909842070000*X1^12+ 40.04951357841491700000*X1^13+ -9.34289753378470780000*X1^14+ -42.32602667808532700000*X1^15+ 5.65262481257468610000*X1^16+ 24.40732628107070900000*X1^17+ -1.51321717932046340000*X1^18+ -5.89714848995208740000*X1^19+	X4= 0.25874558828494687000+ 0.00116615943989017980*X1^1+ -0.09370305367139275500*X1^2+ 0.04326088697416707900*X1^3+ 0.22944046422412612000*X1^4+ -0.93365782871842384000*X1^5+ -2.57513941204121010000*X1^6+ 8.71797290444374080000*X1^7+ 15.23820672201236400000*X1^8+ -43.85258543491363500000*X1^9+ -50.98315056676892700000*X1^10+ 128.42374897003174000000*X1^11+ 99.67723387047044500000*X1^12+ -225.93347322940826000000*X1^13+ -113.13078427802429000000*X1^14+ 235.13924753665924000000*X1^15+ 69.18502820802859800000*X1^16+ -133.33349180221558000000*X1^17+ -17.67463392936110700000*X1^18+ 31.75831666588783300000*X1^19+	X4= 0.25872414294541740000+ 0.00218952529007765410*X1^1+ -0.28711437892281433000*X1^2+ -0.11474610300501809000*X1^3+ 0.09467011441657291500*X1^4+ 5.82031850516796110000*X1^5+ 0.56867189035256160000*X1^6+ -174.21119821071625000000*X1^7+ 4.15725120907882230000*X1^8+ 2847.46576309204100000000*X1^9+ -278.86300692148507000000*X1^10+ -27512.49142456054700000000*X1^11+ 3717.06452841684220000000*X1^12+ 161644.86083984375000000000*X1^13+ -21917.94077126681800000000*X1^14+ -567304.79101562500000000000*X1^15+ 61310.82944253087000000000*X1^16+ 1093264.07617187500000000000*X1^17+ -66382.15657681226700000000*X1^18+ -889763.74609375000000000000*X1^19+
X4			
Estimated function of X5	X5= -0.00291216222649381960+ 0.64867459781227588000*X1^1+ 0.04365262278588488700*X1^2+ -0.23647712157747591000*X1^3+ -1.35425623692572120000*X1^4+ 3.78865170210141860000*X1^5+ 16.48487353324890100000*X1^6+ -25.01537586234030600000*X1^7+ -98.04673397541046100000*X1^8+ 92.79775447481188200000*X1^9+ 326.88679599761963000000*X1^10+ -192.81265069143592000000*X1^11+ -636.72084188461304000000*X1^12+ 224.12265730113359000000*X1^13+ 718.24755907058716000000*X1^14+ -136.98606110438686000000*X1^15+ -434.69468307495117000000*X1^16+ 34.89437117650823000000*X1^17+ 108.77539741992950000000*X1^18+	X5= -0.00156830864398216360+ 0.66737813741740326000*X1^1+ -0.20871253625955433000*X1^2+ -1.32277924744723660000*X1^3+ 6.79747200198471550000*X1^4+ 22.01213340540005000000*X1^5+ -81.69792529940605200000*X1^6+ -160.22442125098451000000*X1^7+ 489.64396274089813000000*X1^8+ 620.90964811027334000000*X1^9+ -1633.90411281585690000000*X1^10+ -1355.63317866927900000000*X1^11+ 3180.45693492889400000000*X1^12+ 1675.05565517320060000000*X1^13+ -3588.04350376129150000000*X1^14+ -1094.70251322849530000000*X1^15+ 2172.19306564331050000000*X1^16+ 294.72894110489710000000*X1^17+ -545.88428068161011000000*X1^18+	X5= -0.00286740150323794300+ 0.64842448632645633000*X1^1+ 0.06563797454873565600*X1^2+ -1.23104680830800820000*X1^3+ -5.01435196027159690000*X1^4+ 26.79619901393743900000*X1^5+ 285.35144710540771000000*X1^6+ -726.24347742414102000000*X1^7+ -8435.74569702148440000000*X1^8+ 11439.42840493843000000000*X1^9+ 139541.98205566406000000000*X1^10+ -112706.41110163927000000000*X1^11+ -1372194.00341796880000000000*X1^12+ 701381.51682734489000000000*X1^13+ 8199799.37890625000000000000*X1^14+ -2657743.19020271300000000000*X1^15+ -29211744.45312500000000000000*X1^16+ 5565915.56249618530000000000*X1^17+ 57045139.81250000000000000000*X1^18+ -4919005.74142837520000000000*X1^19+ -47001062.00000000000000000000*X1^20+
X5			
Estimated function of X6	X6= 2.89526127138450340000+ -0.00659959714539581910*X1^1+ 1.05570823063877750000*X1^2+ -0.18524028745014220000*X1^3+ -6.31489422979119030000*X1^4+ 2.72991794068366290000*X1^5+ 46.29017527471736300000*X1^6+ -19.13156626373529400000*X1^7+ -163.80979793266837000000*X1^8+ 67.80064900219440500000*X1^9+ 302.79609119648160000000*X1^10+ -125.60299882292747000000*X1^11+ -279.37539797732717000000*X1^12+ 118.05200903117657000000*X1^13+ 101.57023962263294000000*X1^14+ -45.09170855581760400000*X1^15+	X6= 2.88418800220961820000+ 0.16979065437044483000*X1^1+ 2.41716662746390610000*X1^2+ -8.24893121444620190000*X1^3+ -33.18561544813537300000*X1^4+ 107.87552714906633000000*X1^5+ 241.93556902189965000000*X1^6+ -614.07626396417618000000*X1^7+ -832.88981423954260000000*X1^8+ 1776.00263896584510000000*X1^9+ 1465.67227544617090000000*X1^10+ -2725.72459515929220000000*X1^11+ -1272.83054430824970000000*X1^12+ 2115.37847287952900000000*X1^13+ 432.10185412653846000000*X1^14+ -653.76943070068955000000*X1^15+	X6= 2.89872152014899020000+ -0.01385549449634027000*X1^1+ 0.97876272603032277000*X1^2+ 0.36579956805508118000*X1^3+ -0.99048284207492276000*X1^4+ -4.57714397413656120000*X1^5+ -10.86867282170240000000*X1^6+ -14.21063726395368600000*X1^7+ 5.41359898325275250000*X1^8+ 808.61749762296677000000*X1^9+ 521.00385404010342000000*X1^10+ -7131.31718730926510000000*X1^11+ -2755.07192750513650000000*X1^12+ 25882.72533130645800000000*X1^13+ 4364.86658143642130000000*X1^14+ -34396.09129142761200000000*X1^15+

X6	 residual (H(X1),error) residual plot,The estimated line is X6=H(X1) SD= 0.0040625948 MSE= 0.0000165047 0.0213 2S S 0.0 -S -2S -0.0187 2.89 H(X1) 5.869014	 residual (H(X1),error) residual plot,The estimated line is X6=H(X1) SD= 0.0374242707 MSE= 0.0014005760 0.2526 2S S 0.0 -S -2S -0.2526 2.88 H(X1) 6.938561	 residual (H(X1),error) residual plot,The estimated line is X6=H(X1) SD= 0.0007914545 MSE= 0.0000006264 S 0.0 -S -2S -0.0020 2.89 H(X1) 3.039265