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Editor's Introduction to Volume 2, Issue 2 of Expert Journal of Economics

Simona VINEREAN*

Sprint Investify

In this second issue of the second volume of *Expert Journal of Economics*, we gathered various interesting articles exploring economic objectives particularly related to the energy resources of Azerbaijan, game theory, uncertainty and partially unforeseen events, and elasticity of labor supply. We are appreciative of the opportunity to publish such meaningful contributions to economics knowledge. Further, I present a short description of each article that is published in *Expert Journal of Economics*, vol. 2, issue 2.

Suleymanov and Yusifov (2014) present an overview of the Azerbaijani economy after independence in terms of the economic objectives that involved the energy resources of the Caspian Sea consisting of oil and natural gas reserves. The authors offer a comprehensive SWOT analysis of Azerbaijan's new country strategy and use the resources efficiently, by listing the both internal strengths and weaknesses of economy and external opportunities and threats. There are also certain potential problems for the investment climate in Azerbaijan in terms of high tax rate, bribery, corruption, presence of monopoly, inequality of business agents in the economy. However, the government is adopting certain programs to adjust the imperfections of the economy and their perceived issues with potential external investors. Also, Suleymanov and Yusifov (2014) explore how Azerbaijan is facing a hazard of 'Dutch syndrome', which is expressed as the collapse of the other regions or sectors as a result of the rapid development, and a consequent dependence, of a specific region or a sector, and possible remedies for this situation.

Andrejs Jaunzems (2014) in his article entitled *Pretension Strategy in the Surviving Game* discusses the important role of game theory for the investigation of individuals' interaction strategies and understanding of social and economic processes. The author presents the surviving game of three agents under competitive premises. Also, the paper establishes that the shape of Nash equilibrium in a specific way depends of the surviving probabilities of the agents. Jaunzems (2014) assesses that the most valuable result of this research is the solution of Martin Shubik surviving game presented in geometrical form. Martin Shubik's statement regarding the "survival of the weakest" in circumstances of asymmetric information gets transformed into "the surviving agent is not weakest one, but only pretends to be weakest one".

In *Partially Unforeseen Events. Corrections and Correcting Formulae for Forecasts*, Harin (2014) explores the hypothesis of uncertain future and the partially unforeseen events that exert an influence on forecasting. The unforeseen events usually have dramatic impact on the outcome of a specific situation. Thus, Harin's article aims to shed some light on the consequences of such events. The author offers valuable mathematical insights on the consequences of the hypothesis of uncertain future, corrections of forecasts, and corrections of the formulae for forecasts. Harin explains how assuming a future partially unforeseen event which has at least one unknown characteristic and preparing preliminary corrections of a forecast, if the

* Correspondence:
Simona Vinerean, Sprint Investify, The Bucharest University of Economic Studies, E-mail address: editor@expertjournals.com

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event were to happen then the forecast would require only simple corrections and it would be much easier and faster than constructing a new forecast.

Malakhov (2014) elaborates on one of the implications of prospect theory, namely the negative elasticity of the individual labor supply which can be better comprehended using the Slutsky equation in relation to the common consumption-leisure choice. Empirical evidence presented by Malakhov shows that leisure is a net complement for an important part of consumption.

References

Suleymanov, E., and Yusifov, S., 2014. Problems Encountered during the Transition to Market Economy in Azerbaijan and Solution Attempts. *Expert Journal of Economics*, 2(2), pp. 45-54

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Problems Encountered during the Transition to Market Economy in Azerbaijan and Solution Attempts

Elchin SULEYMANOV* and Sabuhi YUSIFOV

Qafqaz University, Azerbaijan

After re-gaining its independence on 18 October 1991, the Republic of Azerbaijan started the transformation to the market-based economy and the integration into the world economy. The country's oil and natural gas reserves have been considered the main source for financing a range of government programs for reforms. On the one hand, these reserves had to be used effectively; on the other hand, there was a huge demand for foreign investment for extraction. To this end, Azerbaijan has signed "Contract of the Century" in 1994. Although Azerbaijan has wide oil and natural gas reserves, it has faced a number of difficulties in its transition path. This study analyzes these problems and reforms for solving them. One of the types of the problems related to the economic structure of the former Soviet Union: disruption of the economic ties between the republics resulted in a decline of production, high levels of unemployment and prices and consequently led to an economic recession in all of the republics. Another set of problems related to the lack of sufficient institutional bases to transform to the market economy. Moreover, internal conflicts between the political parties and groups for having authority as well as political chaos in the republic can be considered other serious problems during the transition period. Furthermore, Karabakh war and occupation of 20 percent of the Azerbaijani territory by the Armenian military forces had made the situation extremely complicated. Despite all of these extremes, Azerbaijan transformed to the market-based economy decidedly and even became one of the fast growing countries of the world. Even in 2013, with the GDP growth rate of 5.6 percent, Azerbaijan was a leader among growing economies. In parallel with this significant economic development, there is still a need for some socio-economic and institutional reforms in order to get a well-functioning market-based economy in Azerbaijan.

Keywords: *Azerbaijan, oil, natural gas, informal sector, energy resources, oil dependency*

JEL Classification*: P21*

1. Introduction

After gaining its independence in October 18th 1991, Azerbaijan has launched a number of reforms to liberalize it economy and realize a smooth transition to market economy. Energy sources such as gas and oil

* Corresponding Author:
Elchin Suleymanov, PhD, Qafqaz University, Faculty of Economics and Administration, Khirdalan city, 120 Hasan Aliyev, Baku, Absheron, Azerbaijan Republic

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reserves in the basin of the Caspian Sea were main instruments in achieving these targets. Efficient utilization of these resources served as a pre-requisite of these economic goals and hopefully Azerbaijan succeeded in that. Since 1994 Azerbaijan has started extensive utilization of Caspian energy resource in the name of economic, political and social reformation, regeneration, growth and independence. A number of resolutions were made and followed by contracts and project in order to produce oil and gas and distribute them to world market (Suleymanov, Zeynalov, 2012).

Other newly independent countries of Caspian Basin, Kazakhstan and Turkmenistan also decided to join this process by utilizing their potential oil and gas reserves with the aim of relieving pains of transition period to reorganize their economies. However existing pipelines had limited capacity and did not coincide with new routes to open up to world energy market, because they were set according to Soviet Union's planned economy and covered only Union countries. The Caspian Basin gained great importance immediately after the collapse of the Soviet Union and proclamation of independence by Caspian coastal countries, namely Azerbaijan, Kazakhstan, and Turkmenistan. Improvements in energy resources of the Caspian Sea became unavoidable for these countries for reformation of their economies and achieving integration to world economy. Especially foreign capital know-how were needed for efficient development of resources. In this regard, the three states invited western firms to invest in the region for energy resource extraction.

Azerbaijan and Kazakhstan wanted their oil revenues to flow rapidly to the country (Aras, 2005). Being one of the world's most important energy sources oil has a wider field of use among other sources of energy. Expenditure on extraction, transportation and distribution of oil comprise the biggest cost of petroleum products.

For all three countries methods for efficient use of the oil proceeds need to be worked out in order to improve non-oil sector and decrease the risk of oil dependency in the long run and achieve rapid economic growth and improvements in real income (Aras, 2008). Some of the macroeconomic indicators of Azerbaijan are displayed in the following table.

Table 1. *Main socio-economic indicators of Azerbaijan*

	2000	2008	2009	2010	2011	2012
Population (thousand)	8 114.3	8 897.0	8 997.6	9 111.1	9 235.1	9 305.1
Total GDP (mln $)	5 272.8	48 852.5	44 297.0	52909.3	63402.5	64412.5
GDP per capita ($)	662.9	5 603.3	5 018.2	5922.0	7 003.4	7 206.5
Total investments (mln AZN)	967.8	9 944.2	7 724.9	9905.7	12 776.4	13 776.4
Foreign trade volume (mln $)	2 917.3	54 926.0	20 824.5	27960.8	36 326.9	36 529.9
Exports (mln $)	1 745.2	47 756.0	14 701.4	21360.2	26 570.9	26 670.9
Imports (mln $)	1 172.1	7 170.0	6 123.1	6600.6	9 756.0	9 859.0
State budget revenues (mln AZN)	714.6	10 762.7	10 325.9	11403.0	15 700.7	17 360.7
State budget expenditure (mln AZN)	764	10 774.2	10503.9	11765.9	15 396.2	17 796.2
Personal income (mln AZN)	4 047.3	20 735.4	22 601.1	25607.0	30 633.5	32 633.5
Average salary (mln AZN)	44.3	274.4	298	331.5	363.1	423.1
Official unemployment	43 739	44 481	41 100	38 966	38 330	37 330
CPI (ratio to the previous year)	101.8	120.8	101.5	105.7	107.9	105.9
PPI (ratio to the previous year)	129.7	123.4	80.6	130.5	133.5	123.5

Source: NSCAR 2012

Caspian energy resources were a motivating factor and in parallel with foreign capital inflows to the country, the international financial institutions started to increase the loans to Azerbaijan. During the last period of the Soviet Union, the process of collapse caused chaotic environment where distribution of state property and social wealth for the purposes of personal use was commonplace. From this point, bribery and corruption inherited from the Soviet system of society was one of the severe social issues. Although some changes were made to deal with bribery and corruption, the presence was long felt widely in the general population. Formal and informal tax systems were available. The regulatory role of informal taxes, was greater than the regulatory role of official duties (Aras, 2005).

2. Overview of the Azerbaijani Economy after Independence

After the declaration of independence most important resources in reforms to achieve Azerbaijan's economic objectives for the realization of the country's free market economy were energy resources of the Caspian Sea consisting of oil and natural gas reserves.

Since 1994, Azerbaijan's oil and natural gas agreements with international companies and common activities in this context have allowed the re-establishment of the infrastructure in the energy sector through the introduction of advanced modern technologies. For example, with the realization of the first stage of the "Contract of the Century" not only the problem of the evaluation of oil stocks in Azeri - Chirag - Guneshli project was successfully solved, but also Azerbaijan has benefited from new oil production technology in the Caspian Sea and modern systems for export of Caspian oil to world markets were established. Azerbaijan has gained the opportunity to renew its energy industry assets by transferring the modern technology to the country and has established institutions and organizations that contribute to the development of oil and gas industry worldwide. Thus, modern research, oil and gas extraction, processing and transportation technologies have been exploited extensively.

In this regard, projects and agreements related to the production and export of Caspian energy resources opened a new page in the history of Azerbaijan's energy industry. In today's global competitive environment, the new state administration of Azerbaijan determined variety of economic development programs, development policies and strategies in order to realize the transition to market economy. It is of strategic importance to make a conscious and systematic analysis of the country's resources and potential opportunities and make a SWOT Analysis in order to develop a new strategy and use the resources in the best way. From the perspective of investing in different sectors, a preliminary strategic step must be to conduct a non-empirical analysis of the country's economy in terms on the macroeconomic, sectorial and non-economy spheres. (Aras, 2005)

Strengths of Azerbaijan's economy in terms of macroeconomics can be listed as following:

- Presence of stable macroeconomic environment in general and especially in terms of price stability: The last 20 years have seen a single digit inflation rate. Prices have remained very stable.
- Low proportion of external debt to GNP: macroeconomic instability was observed in the first years of its independence in the country, namely until 1995 due to the political, economic instability, as well as Nagorno-Karabakh conflict with Armenia. However after the political stability was reached in the country in 1996, and after implementation of certain economic policies, instability left itself to stable growth. Thus, favorable macroeconomic indicators have dominated in the country for nearly 20 years (Suleymanov, Zeynalov, 2010). The following is public debt as a percentage of GDP:

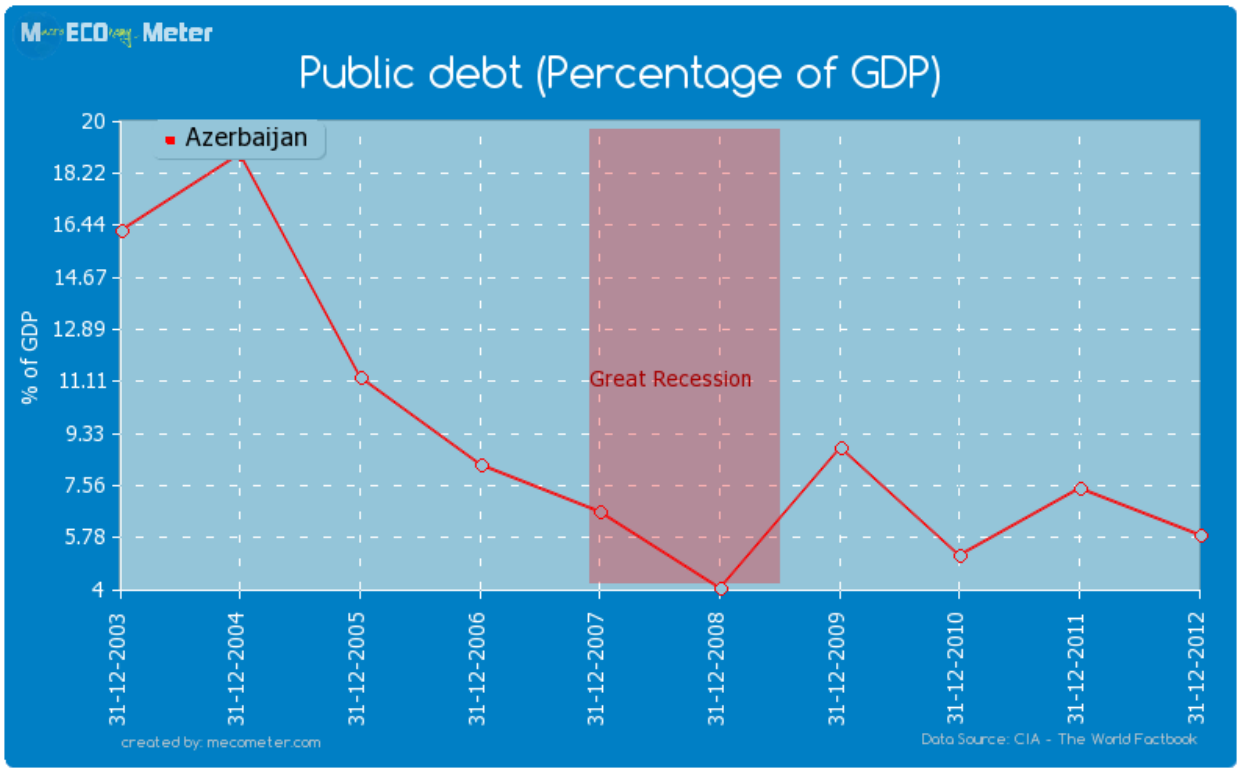

Figure 1. *Public debt as a percentage of GDP (2003-2012)*
Source: http://mecometer.com/whats/azerbaijan/public-debt/

- Stable exchange rate: Although, from time to time Manat appreciated against the dollar and euro, in the long run it showed a stable trend. Azerbaijan left "Ruble Zone" in 1992, and since then the national

currency, Manat, was put in circulation. In the first stage, the Manat circulated together with Russian ruble in the country until 1994 when Manat has become the only official currency. As significant portion of the imports were made with ruble, Manat began to lose value against the ruble in 1993 when prices in importing countries, especially in Russia began to rise as a result of price liberalization. On the other hand, the crisis in the banking sector in 1993-94 and negative expectations regarding the value of Manat caused Manat to depreciate against the currencies of other countries. Manat has displayed stable a trend since 2000. On 1st January 2006, as a result the denomination, nominal value of Manat was reduced 5000 times. One Manat was determined to be worth about 1.2 U.S. (Aras, Suleymanov, 2010). The rate of Manat has not changed much for nearly seven years except for very short exchange rate volatility and at times Manat has gained value compared to the dollar. Exchange rates remained stable as economic and political stability provided in the country. According to the World Bank data exchange rates (AZN to USD) were on average 0.80 between 1994 – 2003 and 0.79 between 2009 – 2013

Table 3. *Official exchange rate (US$ per AZN, period average).*

	1994-1998	**1999-2003**	**2004-2008**	**2009-2013**
Azerbaijan	0.80	0.80	0.79	0.79

Source: http://data.worldbank.org/indicator/PA.NUS.FCRF/countries/1W-AZ?display=default

- Presence of cheap labor: Due to the high rate of unemployment in the country, wages are low. According to data of 2013, the minimum wage in the country is 105 Manat, the average wage is around 360 Manats. Payment for labor as a factor of production is advantageous in terms of total cost. This is another factor that encouraged investment in the country.
- Non-economic Strengths can be listed as follows:
- Education level of the population: Despite difficult conditions preceded by decline in the economic situation, people's interest in education was still unchanged as a continuation of the Soviet tradition. Literacy rate in the country is 98%, of which 60 % is highly educated. In each country, primarily for economic development, political stability and this stability will continue to be the managers. Beyond doubt that, each country needs primarily political stability for economic development and strong-willed administrators to pursue these goals. Political stability will serve as an indicator of uninterrupted execution of the economic programs and give confidence to the markets.
- There was still need for continued political stability in Azerbaijan gained when Heydar Aliyev came to power. Uncertainties were drawn to the minimum level with election of President Ilham Aliyev and it increased positive expectations.
- Strategic location: Asian countries are important for the U.S. and European countries because of their strategic location in the corridor to Europe and as well as their bordering with Iran and Russia. Azerbaijan added a new link to the chain of global economic relations with the agreement signed in 1998 in Baku which envisaged transfer of historical Silk Road to 21st century as a communication, transportation and energy transport corridor. Silk Raod served as a historical route for caravans carrying the riches of the East to the West for centuries passed by, Tajikistan, Georgia and Armenia, as well as five of the Turkish Republic. The weaknesses of the national economy from macroeconomic perspective are as follows:
- The unemployment problem: Official statistics in Azerbaijan express that the unemployment rate is around 1.5%, but with different calculations and information provided by international organizations, and in the opinion of experts, the ratio is between 16-25%. Therefore, unemployment in the country is high. One of the reasons for high unemployment level, is that around 1 million people were exposed to forced migration after the occupation of Karabakh by Armenians. On the other hand, closure of many companies which were not competitive enough and compatible with modern production, constitute the most important ratio in the number of unemployed.
- Migration of educated and skilled workers abroad: Due to lack of employment opportunities for educated and skilled workforce, many of them emigrated overseas. The majority of skilled and educated workforce moved to Russia, Turkey, Iran. It is noteworthy that among those who left the country in this way, the higher proportion is made up of young and middle-aged people. This “Brain drain" process has increased more in recent years.
- The share of oil and oil revenue in exports is around 90%. On the other hand, the 75% of budget revenues are derived from petroleum and petroleum-related fields. Thus, as the budget is dependent on the oil revenues and oil revenues in turn are dependent on the world market conjuncture, Azerbaijan has confronted with danger of the 'Dutch Disease' (Sabiroglu, 2001). The following table displays the

dynamics of transfers from State Oil Fund to state budget between 2003-2012.

Table 2. *Transfers from SOFAZ to state budget (2003-2012).*

Year	Transfers from State Oil Fund (mln AZN)	Share of the transfers from State Oil Fund in total budget revenues, %	Share of the transfers from State Oil Fund in GDP, %
2003	100.0	8.2	1.3
2004	130.0	8.8	1.5
2005	150.0	7.2	1.2
2006	585.0	1.1	3.2
2007	585.0	9.7	2.1
2008	3800.0	35.3	9.5
2009	4915.0	47.6	14.2
2010	5915.0	51.9	14.3
2011	9000.0	57.3	18.0
2012	9905.0	58.1	19.8

Source: htttp://budget.az/budget/en/main?content=526

- Slow processing in the privatization of large enterprises.
- Although bureaucracy and corruption declined each year, it is still important problem of the transition process. On the other hand, there is injustice in the distribution of income. In particular, the economic situation of workers in oil and petroleum-based sector is very good while income level of workers in other sectors is very low. Consequently, certain proportion of the population is headed for luxury consumption. The middle layer is not yet formed in the country.
- Monopoly and unfair environment for competition: One of the major problems faced by businesses in the country is the existence of monopoly or unfair competition environment. The existence of monopolies in certain areas of the private sector hampers the development and efficient use of resources. It also causes chaotic increase in the prices from time to time.
- Fluctuations in exchange rates due to lack of confidence in the domestic currency. In the first years of independence, the inflationary process and devaluations in Manat, as a result of the political and economic crises caused people's distrust against the domestic currency.
- The informal economy: the ratio of informal economy in the country's GNP is high. In fact, Azerbaijan ranks high among CIS states the volume of the informal economy. According to official data of the informal economy corresponds to somewhat 18%-20 of GDP. However, in some evaluations, the informal economy is calculated to be greater than the economic activity. According to World Bank experts, the volume of the informal economy is more than 60% of GDP. This ratio which is considered to be off the record corresponds to 8-10 billion dollars. Some experts argue that this ratio is 80%. Despite different figures, in general, high informal economic activities are observed.
- Lack of adequate infrastructure and old technology: Agriculture and livestock watering systems are inadequate. On the other hand, technology needed for agriculture and animal husbandry is both old and inadequate. Facilities essential for tourism, such as transportation and accommodation are incomplete and outdated too. Also, on the industrial sector, inadequate and outdated technology infrastructure is observed. This does not allow producing and manufacturing good quality and cheap products for export to the world market. High dependency on oil and natural gas production cause lack of interest in the development of other sectors. After the banking crisis in the first years of independence, banking sector located in the center of the financial sector has not developed yet. Failure to carry out the privatization of large-scale enterprises has been the most important factor in the underdevelopment of the securities market. Due to low levels of income, the development of the insurance sector has been hampered.
- Existence of sectored monopoly: Presence of monopoly in some sectors has an adverse effect on the price and the quality of the products produced in those sectors, and is an obstacle to the efficient use of production recourses.
- Dutch Disease Hazard: Since exports and foreign investments are oriented towards oil and natural gas production, export products comprise mainly oil and natural gas and budget is also oil revenue dependent, it is obvious that the country’s economy is reliant on oil and gas industry. This dependency has further increased since 2006, with the boom in the oil exports.
- Imbalance in regional development: On the other hand, as most economic operations and tax revenues are collected in or around Baku, most of firms and companies agglomerated in Baku, the regional economic

structure displays dependence from Baku.

- After independence in relation to the production of oil and natural gas, the energy sector has been at the center of attention of foreign and domestic investors. Again, skilled labor was also trying to find more jobs in this sector. As a result of accumulation of both investment and skilled labor in a single sector, serious progress has not been made in other sectors.
- Following are non-economic weaknesses:
  - Regional transport barrier: after the Armenian invasion, 130 km of railway transportation that joined Nakhchivan and other regions of the country has been occupied. Therefore the road and rail transportation between Nakhchivan Autonomous Republic of Azerbaijan and other regions cannot be made due to Nagorno-Karabakh territory, occupied by the Armenians. Today passenger and freight transportation to Nakhchivan can be done through Iran.
  - Mindset change: In the process of transition from the Soviet regime to democracy and free market economy, change of mentality has not yet finished and is rather incomplete. The problems are still evaluated with a view similar to the Soviet system. This in turn implies settlement of market economy in the country. Mindset change is a long process, of course and requires time.
- Threats to economy:
  - Oil price fluctuations. Fluctuations in oil prices are the most economically significant threat.
  - Continuation of the problem of Nagorno-Karabakh: Nagorno-Karabakh has not escaped from Armenian occupation yet. There are still conflicts at times in borders. If the problem is not resolved with dialogue there is a tendency to get the occupied lands by force which means the country is likely to be faced with war. This problem is causing a deterrent effect from the point of foreign investors, especially to invest in non-oil sector. Up to 2012, 70% of investments comprising a total of 100 billion dollars are foreign investment. From foreign investments, only 30% of the direct investments were invested in non-oil sector. There were no serious investments in non-oil sector by domestic entrepreneurs.
  - Border countries: Occasional political and social problems in Georgia, problems in Dagestan Autonomous Republic arising from Chechnya and Iran's political structure and its tough relation with international community has also had an adverse effect on foreign and domestic investors. Investment held in Azerbaijan, especially large-scale investments, will be made taking into account not only domestic market potential, but also other countries in the region. When Azerbaijan's border countries experience some problems from time to time, it leads to problems at the border and negatively impact foreign trade. Tense relations in the region and sensitive policies led by the country also adversely affect the foreign trade (Aras, 2005).
  - -The status of the Caspian Sea: Inability to determine the status of the Caspian Sea caused negative effects on the country's economic development. Because of this controversy, some of the oil companies do not want to invest in oil production in these beds. Along with petroleum and natural gas reserves, very important fish and caviar resources of the Caspian Sea cannot be used efficiently and involved in economy. On the other hand, the problem of environmental pollution in the Caspian sea and harmful externalities have their negative effects on the economy. Caspian Sea status issue is the obstacle to the development of sea transport.
  - The absence of outlet to seas and oceans: In the absence of the country's outlet to seas and oceans it is not possible to integrate into intercontinental transport and this deprives country form benefitting from maritime transport.

3. Problems Encountered during the Transition to Market Economy in Azerbaijan and Solution Attempts

Within the framework of SWOT analysis of Azerbaijan's economy, both internal strengths and weaknesses of economy and external opportunities and threats faced by the country economy were listed.

The first strategic step by foreign and domestic investors on investment decisions will be made by looking into strengths and weaknesses of the domestic economy and by taking into account the opportunities and threats of economy seen from outside.

It would be appropriate to invest in the area where strengths are in alignment with the opportunity. Only after this step they will be able to decide whether to further invest in terms of sub-sector, products and regions. After the first step it is necessary to get legal and diplomatic support and assurance, as well as build up individual relationships.

The strengths and weaknesses mentioned in the analysis above are important to determine which sectors, how and in what manner to invest in order to achieve economic development of the country. On the other hand, threats and opportunities of the country economy are important for political forces to decide on which programs, projects to implement in order to achieve economic development and integration into the world economy (Aslanlı, 2004)

State administration should make and implement right decisions by harmonization of strengths and opportunities. On the other hand, strategic decisions and policies should be developed by transforming weaknesses into strengths and threats to opportunities. (Bağırzade, 2009)

The government should take more realistic, steady and reassuring steps and provide incentives in the creation of the necessary investment climate. It should identify factors threatening economic activities, adversely affecting the investment climate, should determine political and social policies to relieve weaknesses and take actual steps in accordance with the policies. Azerbaijan has trading partnership with more than 150 countries, is a member of many international and some regional organizations. The oil contracts and international support to these contracts, that support, the oil companies investing in the country give confidence and establishes high investment environment in the country.

- Early transition to a free market compared to other countries in the region. Azerbaijan is much ahead of the newly independent republics in terms of the transition to a free market economy in and carrying out the legal and institutional infrastructure.
- The country which attracted most investments among newly independent countries. Most foreign investments in the region among other countries gained independence is seen in Azerbaijan. This would be a positive indicator for investors in other sectors.
- Being among petroleum and natural gas exporter countries. As a result of work done after the discovery of the new oilfields work done; Azerbaijan joined the chain of oil exporting countries since 2005 and gas exporting countries since 2006. This was the most important opportunity for attracting foreign investment and integrating to world economy.

Non-economic Opportunities are:

- Being on the trade routes and historical intersection: Azerbaijan is geographically located on very important historical New Silk Rode transit route which links the two continents and three sea basins.
- Progress made in the transition to democracy: Azerbaijan is much ahead of the newly independent countries in the region in terms of transition to democracy. This feature allows the country to become a main market for investments in the region.
- Existence of a more stable and democratic government among the countries of the region.
- The political stability and the government's democratic attitude gained after Heydar Aliyev came to power, was an effective factor for investors to prefer Azerbaijan in the region.

Imperfections on Investment Climate:

In terms of the availability of investment climate in Azerbaijan, number of negative factors such as the lack of the necessary legal arrangements, the high rate of tax, bribery, corruption and the presence of monopoly are among the issues of concern for entrepreneurs. Competition policy is considered as one of the essential elements in countries in transition. Lack of the same conditions in the economy hinders the activities of businessmen, thus preventing economic development. Some measures have been taken by the government of Azerbaijan for the removal of inequality in the economy. The adoption of the program to fight against corruption is such an example. Because of high tax rates, the private sector tends to hide economic activities. A large part of private enterprises are giving wages to the majority of their staff unofficially. In fact, informal economy is perceived as illegal production and sales of weapons, narcotics and pornographic products. However in Azerbaijan, informal economic activities are carried out mainly in the areas of the simple production, trade and services. (Bağırzade, 2007). In some evaluations made, the informal economy is recorded to be greater than legal economic activities. 80% of foreign investment in Azerbaijan's seems to be in oil sector. It is worrying to observe that sectoral investments also mainly fall under this category and a significant part of GDP is formed from oil revenues. Price changes in the world market due to the one-sided development of the petroleum sector also cause fluctuations in GDP. This situation was clearly seen in the 1998 Russian crisis.

Briefly, Azerbaijan is faced with a hazard of "Dutch syndrome," which is expressed as the collapse of the other regions or sectors as a result of rapid development of a specific region or a sector.

When economic development is based on the export of natural resources alone, this means only a small group of the population benefit from them, and depletion of these reserves result in a decrease in the living standards of the majority which contradicts to the principles of social justice. Therefore, when the economy relies only on natural resources sector, this would undermine the independency of the countries in both internal

and external relations.

High dependency of economy on energy sector will cause negative results on the socio-economic situation. Circulation of potential and skilled labor in the energy sector will adversely affect the development of other sectors. Spending of oil revenues in the purchase of imported goods, especially if consumer goods are imported rather than capital goods, the country's trade balance will turn to negative. In order to defeat the risk of "Dutch syndrome", policies such as strengthening foreign investment flows to Azerbaijani economy, and provision of all necessary conditions for fulfillment of these investments must be among the main objectives for the government. (Bulut, Suleymanov, 2012) The first step should be provision of favorable climate for investments. To realize this first of all non-commercial risks must be eliminated, and regulations on the tax rates, the tax collection system and ethics, and necessary arrangements for administrative and legal regulations for securing foreign capital and for optimization of the tax and customs systems should be made. Besides the Dutch Disease, there are several other caused by foreign capital: the country's increased external debt; diversion of foreign investment into fast revenue-generating activities such as raw material and service areas; presence of very little investment in the country's manufacturing industry; aggregation of almost all investment in Absheron; negligibly small amount of foreign capital in other regions. Despite the fact that the advice and borrowings from international financial institutions played a major role in macroeconomic stabilization in the country, very little progress is observed in the increase in efficiency, levels and yields of production in the real sector and also in socio-economic problems such as social security. One of the major problems faced by operators in the country is tendency to monopoly. Monopolization of certain areas by companies or businessmen hinders the development of private entrepreneurship and even leads to closure of some companies. Monopoly and unfair competition is hampering the development of the private sector in Azerbaijan. The existence of monopoly results in chaotic rises in prices.

4. Conclusion

Briefly, Azerbaijan is facing a hazard of "Dutch syndrome," which is expressed as the collapse of the other regions or sectors as a result of rapid development of a specific region or a sector. Even beyond dependence on one sector, Azerbaijan is facing with the danger of dependency on one region. The conclusion drown from analyses of Azerbaijan's economic development after independence is that, in the 1991-1996 period there was significant progress in almost all parameters. Starting from 1996 stability was ensured and there were positive developments further with the exception of negative effects of Russian (1998) and (2008) world economic crisis.

In particular, with the rapid development of oil and natural gas industry in the 2000s, a very large improvement in basic economic parameters were recorded, and it is expected to continue in the coming years. Thus, despite all the positive and negative processes in economy after independence, between 1991-2013 GDP volume (in other words, Azerbaijan Economy size) increased about 120 times, per capita GDP volume raised 400 times, state budget expenditures 280 times, the total foreign trade volume 30 times, exports 10 time, while imports increased 10 times. Today with all the basic quantitative parameters on the economy, Azerbaijan is a leader in the South Caucasus. Azerbaijan's foreign debt was U.S. $ 5.688 billion at the beginning of 2013. This amount corresponds to 13.6% of GDP. By the amount of external debt per capita, Azerbaijan is ranking in the middle compared with the CIS countries. But despite all the positive developments and government strategies and policies implemented after independence in Azerbaijan, there are still certain problems experienced in the economy. The most important problems can be listed as follows:

- High dependency of the economy on the oil sector,
- Absence of economic growth model generating employment,
- Regional inequalities, inefficiency of the agricultural sector,
- Socio-economic problems endangered by the occupation of Nagorno-Karabakh and the surrounding regions by Armenia,
- High interest rates, high tax burden, underdeveloped public services,
- The low rate of incorporation, weak corporate governance tradition,
- "Cash economy" is common, artificial monopoly, informality, corruption and various infrastructure problems exist.

The following policy measures may be effective for the solution of the current problems in the economy:

- Sustainable economic development, increasing employment, poverty reduction, reducing injustice in income distribution, reducing regional inequalities and ensuring horizontal balance between regions,

following government policies in the framework of government's 2008-2012 and 2013-2015 regional development programs which aim development of non-oil sector.

- The applicability of the laws and in the process of transition to market economy and benefiting from the experience of developed countries of the world. The country should transform from natural resources-based economic growth model to high technology and efficiency-based model of economic growth. In order for this, it is important to promote the new technology transfer to the country, to build and develop techno-parks, to implement most perfect models of science and economics cooperation, to establish optimal sized economic agents and apply modern production and management systems that lead to improved efficiency in resource allocation and productivity, to improve quality of education, particularly in technical fields.
- Efficient utilization of approximately $ 50 billion accumulated in the Oil Fund in 2013. These resources must be used effectively and efficiently in both local and offshore investments in order to further increase and guarantee post petrol period.
- In order to reduce poverty and income inequality, existing policy mechanisms for poverty alleviation need to be developed and fiscal policy instruments in reducing income inequality should be used in an active way.
- To ensure the continuity of policies against inflation and minimize its negative impact on the economic system.
- Influence of Interest policies must be promoted in order to divert informal savings into investment in formal sector. From this point the gap between average interest rates on deposits and average return of capital in informal sector must be minimized. Interest on investment loans must be low enough in order to encourage investment activities in the formal sector.
- It is essential to ensure continuity of foreign exchange earnings in the long run which are important elements of macroeconomic stability in the country. Policy measures need to be taken, with the purpose of reducing dependency of export incomes on oil, by increasing the export potential of non-oil sector.
- In order to ensure budget balance sustainability and increase effectiveness of socio-economic functions of budget, there is a need to reduce its dependency on oil revenues and raise the ratio of continuous source of income such as taxes in budget.
- Policies that encourage foreign investment in the non-oil sector should be developed. These policies will boost economic development and limit the effects of informal sector.
- State incentive mechanisms need to be developed for entrepreneurs. In this regard, promoting the expansion of cheap lending mechanisms of National Fund of Support for Entrepreneurship for new entrepreneurs, the development of the incentives mechanism to the agricultural sector to increase the efficiency, the expansion of the state support for entrepreneurs in terms of science and know-how, development of the mechanism of awarding entrepreneurs by the state will be some of effective measures.
- Entrepreneurship infrastructure should be developed. In that regard, the development of the capital and insurance markets, the creation of specific industrial zones, developing necessary infrastructure for consulting services and exhibition industry is important.
- Ensuring the rights and interests of entrepreneurs in the country should be raised further. It is mandatory to raise the level of entrepreneurs' unification, to prevent delays in formal procedures and to protect copyright and intellectual property.
- Necessary policies should be developed to fight against informal economy. In this aspect preparation of state programs to fight against the informal economy, creation of official central institutions, the development of statistical data, stiffening of penalties, removal of legal gaps are important measures to take.

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Pretension Strategy in the Surviving Game

Andrejs JAUNZEMS*

Ventspils Augstskola, Latvia

Presently, we cannot find the scientific analysis that clearly explains the deepest roots of global economical and moral crisis. Because of that many famous politicians, economists, sociologists denote the understanding of current situation as the most valuable attainment. Under traditional influence of the doctrine of spontaneous harmony of egoistic individual behavior many economists believe that competition and private property rights through the markets' price mechanism leads in the long run to the Pareto efficiency equilibrium. At the same time the social and economic reality categorically asks to ascertain the market failure and to revision the classical statements of microeconomics. The perfect competition market has lost its attributes due to dialectics of interactions of agents. The investigation of the interactions strategies of the individuals are based on the game theory, what also helps to understand the role of asymmetric information as a specific market failure factor. In the present paper the Martin Shubik classical surviving game is analyzed and some statements of Herbert Gintis concerning this game are critically appraised. The solution of Martin Shubik game in the original geometrical form is offered. The problem of Martin Shubik "does the fittest necessary survive?" is transformed according the case of asymmetric information in problem "does the pretender survive?", for which the answer "if the agent is not the weakest, but he pretends to be the weakest, than this agent survives with high probability" is offered. The results of the present paper appear to be innovative, not discussed in literature available to the author of the present paper.

Keywords*: game, probability of surviving, interactions of the agent's strategies, Nash equilibrium, asymmetric information, pretension*

JEL Classification: C70, C73

1. Introduction

The view about wide diversity of game theory applications offers, for instance, the content of the Gintis's (2009, p.390) book: "*Game Theory Evolving: A Problem Centered Introduction to Modelling Strategic Interaction.*"

The applications of game theory as tool for decision support is given in the books of Baye (1997, p. 578), Байе (1999, p. 743), Binmore (2007, p. 184), Jaunzems (2008, p. 555; 2009a, p.311; 2009b, p. 360). In paper of Jaunzems (2009c) the analysis of some social economical processes in Latvia with help of game theory is presented. There is no lack of literature and sources, nevertheless in the practice of social and economic analysis in Latvia the game theory introduces too slowly.

* Corresponding Author:
Andrejs Jaunzems, Ventspils Augstskola, Inženieru iela 101, Ventspils, LV-3600

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In Gintis's book (2009, p. 23) under the title "*When Weakness Is Strength*" the model "The surviving game of three agents" offered by game theorist *Martin Shubik* (1954, 43-46) is considered. With help of this model the surviving example which contradicts to the fundamental thesis of Charles Darvin "*survival of the fittest*" is constructed. In this example the highest probability to survive has the weakest player. However this example is too simplified. The deepest investigation of the problem discovers that probability of surviving depends on mutual proportion of agents' strength and from behavior of agents as well. Unfortunately the author of present paper was not able to get original paper of Martin Shubik (Shubik, 1954, pp. 43-46). Therefore the aim of present paper is dispute with Herbert Gintis (2009) and more wide interpretation of Charles Darvin thesis. The limitation of some statements of Herbert Gintis (2009) empirically is proved. In this paper exact definition of surviving game is given, the mathematical model of surviving game under definite comparatively universal assumptions is constructed and with help of model multi-shaped numerical experiments are performed. Besides that the Nash equilibrium of surviving game is identified, the classical surviving game is modified according asymmetric information and instead the theme "*When Weakness Is Strength*" the theme "*When Pretension Is Strength*" is discussed. The authors have never met the mentioned results in literature, therefore comments are welcomed and, if is not proved the opposite, results of the present paper have to be evaluated as innovative.

2. The Definition of the Surviving Game

The definition of the surviving game comes next, with comments.

Three individuals − Ansis, Basis, Casis take part in the surviving game.

At first the sequence of shooting during lottery is determined. As result one from six possible sequences (ABC), (ACB), (BAC), (BCA), (CAB), (CBA) is fixed. The probability of each sequence is 1:6.

The game is extensive − in the each step or act of the game one of players has rights to shot.

Explanation. Let us consider, for instance, the sequence (ABC). The sequence (ABC) means, that Ansis shot the first. Ansis has three strategies: Ansis may shot Basis, Ansis may shot Casis and Ansis may shot in the air.

(1) Let us assume that Ansis shot Basis; we will denote that as A→B. If Ansis hits Basis than Basis exits the game (we will say Basis is shot down or Basis is eliminated) and farther sequence of shooting is (CA). If Ansis miss Basis then the farther sequence of shooting is (BCA).

(2) Let us assume that Ansis shot Casis; we will denote that as A→C. If Ansis hits Casis than Casis exits the game, he is eliminated and farther sequence of shooting is (BA). If Ansis miss Casis then the farther sequence of shooting is (BCA).

(3) Let us assume that Ansis shot in the air; we will denote that as A→O. The farther sequence of shooting is (BCA).

The five another sequences are interpreted analogically.

The definition of surviving game continued. Let us assume that the following probabilities are given.

The probability that Ansis hits is p_A. The probability that Ansis miss we denote as q_A: $p_A + q_A = 1$.

The probability that Basis hits is p_B. The probability that Basis miss we denote as q_B: $p_B + q_B = 1$.

The probability that Casis hits is p_C. The probability that Casis miss we denote as q_C: $p_C + q_C = 1$.

The each of agents is interested to stay alive. The game is finished when only one of the players has left alive. Let us observe that the surviving game may have infinite acts because of possibility that players miss and miss and all three agents or at least two of them have infinite chances of shooting. In the each act the agent, who has the move, chooses the strategy from set of three strategies, if three players are still alive, or only one strategy if two players are remain.

Our main goal is the determination of the Nash equilibrium of this game.

Let us illustrate the Nash equilibrium in the simple, but nevertheless a very pithy case.

Suppose $p_A = 1$, $p_B = 1$, $p_C = 1$ and the sequence (ABC) during lottery is determined. It is easy to see that the triple of strategies (A→O, B→O, C→O) determines a situation that is very stable Nash equilibrium. Indeed, if Ansis shot Basis, Basis is eliminated and the next moves Casis who shots down Ansis and wins. So, Ansis would be unwise if he shots Basis or Casis. The best strategy for Ansis is shot in the air. When the right of move has Basis he judges similarly and makes decision that shot in the air is the best strategy for him. The same decision makes Casis. So the eternal peace exists between these three antagonistic and strong warriours.

The given surviving game associates with popular but wrong thesis about bipolar world as peace guarantee. If only two players Ansis and Basis take part in surviving game and $p_A = 1$, $p_B = 1$, than shooter who has rights to shot first unswerving eliminates the enemy. We recognize here the first move advantage often discussed by military experts.

Conclusion. The antagonistic bipolar world is unstable. The antagonistic tripolar world is very stable.

Important remark. Everywhere further, if especially is not formulated another assumption, we will examine comparatively universal case, namely, we will suppose that $0 < p_C < p_B < p_A \leq 1$. We will say that Ansis is the strongest but Casis is the weakest shooter.

What is the payoff of the player? The payoff of the player could be the surviving probability. However, if we are going to investigate this game analytically with help of mathematical methods we need correct agents' surviving probability definition, what unfortunately we do not meet in Gintis's book (2009) published by Princeton University Press.

Let us illustrate with help of simplest stochastic experiment − dice throw the theoretical difficulties what arise when we are going to definite agents' surviving probability. Try to understand the meaning of following question: "Is the probability that first will come in sight even number bigger than the probability that first will come in sight number one?"

It is possible to calculate, for instance, probability of event "five times throwing the dice the even number will come in sight earlier than number one". In the same time the cardinality of the set of all outcomes when the first comes in sight even number is continuum. Also the cardinality of the set of all outcomes when the first comes number one is continuum.

In order to depict the extensive process of the surviving game graphically as decision-state tree we have to use infinite graph with infinite volume of infinite sub-branches.

2.1. Two Agents' Surviving Game

Before investigation of three agents' surviving game is purposeful to start with two agents' surviving game analysis, what is sufficiently simpler because of each player has only one strategy − to shot enemy.

For instance, let us examine the case when Casis is eliminated and further sequence is (AB). Such a game is possible to depict geometrically as decision-state tree with infinite volume of finite sub-branches and one infinite subbranche.

We are going to define the probability that Ansis survives as sum of infinite geometric series. Let us denote probability that Ansis has eliminated Basis after 2k+1 shoots or earlier with $P_A\{(AB) \mid 2k+1\}$. Let the sign "+" means "hits", the sign "−" means "miss".

Then, for instance, in the case 2k+1 = 5 the event we are interested in consists from the following results of shooting: (A+), (A−, B−, A+), (A−, B−, A−, B−, A+).

We calculate $P_A\{(AB) \mid 5\}$ as follows:

$$P_A\{(AB) \mid 5\} = p_A + q_A q_B p_A + (q_A q_B)^2 p_A = p_A [1 + q_A q_B + (q_A q_B)^2].$$

It is easy to see by analogy that $P_A\{(AB) \mid 2k+1\} = p_A [1 + q_A q_B + ... + (q_A q_B)^k]$.

We calculate the limit of partial sum of the geometric series:

$$P_A(AB) := \lim_{k\to\infty} P_A\{(AB) \mid 2k+1\} = \frac{p_A}{1 - q_A q_B}.$$

Number $P_A(AB)$ will be interpreted as probability that Ansis survives.

The probability $P_B(AB)$ is defined analogically: $P_B(AB) := \lim_{k\to\infty} P_B\{(AB) \mid 2k\} = \frac{q_A p_B}{1 - q_A q_B}$.

As a check on our formulas, note that $P_A(AB) + P_B(AB) = 1$.

Remark. In book published in Princeton University (Gintis, 2009) the probability $P_A(AB)$ has calculated from the recursion equation: $P_A(AB) = p_A + q_A P_A(BA) = p_A + q_A q_B P_A(AB)$.

Solving, we get $P_A(AB) = \frac{p_A}{1 - q_A q_B}$.

Let us mark that this method is heuristic but mathematically incorrect.

In the analysis of three agents' surviving game we will make use of the following theorem about two agents' game what confirms the intuitive suspected connection: the weakest enemy of Ansis, the higher is for Ansis probability to survive.

Theorem. $P_A(AB) < P_A(AC)$ if and only if $p_B > p_C$.

The proof follows from equalities $P_A(AB) = \frac{p_A}{1-q_Aq_B}$; $P_A(AC) = \frac{p_A}{1-q_Aq_C}$.

Consequence. $P_A(BA) < P_A(CA)$ if and only if $p_B > p_C$.

2.2. The Nash Equilibrium of the Sub-Game of the Surviving Game and Nash Equilibrium of the Martin Shubik Game

As it was showed before in general case of surviving game the cardinality of agents' strategies set is continuum. We are going to make important assumption about agents' behavior in three shooters game, which will relieve us the analysis of game and will allow us to get important and pithy interpreted conclusions.

Let us suppose that each of agents chooses his strategy in the very beginning of the game and after that each time when agent has right to move, he acts according this strategy. It means, that Ansis in the very beginning of the game chooses one of three strategies A→B, A→C, A→O, and then utilizes this strategy each time when he has rights to move (naturally, till there are three players in the game). Analogically, Basis chooses one of strategies B→A, B→C, B→O and Casis chooses one of strategies C→A, C→B, C→O.

Taking in account this assumption all together 3×3×3 = 27 triples of strategies exist.

Concrete triple of strategies, for instance, (A→B; B→A; C→O) allows us to follow the extensive process of game so say in the probabilities tongue and analogically as it was did in the case of two players to define agents' surviving probabilities as sum of geometric series.

Given sequence of acts (ABC) and given triple of strategies (A→B; B→A; C→O) determine three surviving probabilities: $P_A(ABC)$, $P_B(ABC)$ $P_C(ABC)$.

The game what corresponds to the concrete sequence determined during lottery we will call as sub-game of the surviving game. We have six sub-games all together, each of that corresponds to one of six possible shooting sequences (ABC), (ACB), ... , (CBA).

The Martin Shubik game is completely defined, if we construct the (27×3)-table represented strategy-surviving probabilities for all sub-games. Thus, in order to investigate Martin Shubik game we calculate 6×27×3 = 486 probabilities. After that, taking in account that initial sequences are stochastic, we calculate the (27×3)-table of mathematical expectations of strategy-surviving probabilities. These table allows us to determine Nash equilibrium of Martin Shubik game and to make different another conclusions.

Let us utilize heiristic method of surviving probabilities determination from the recursion equations offered by Herbert Gintis. (The same results can be get as limits of geometric series.) Than, for instance, we will get the following expressions for Ansis, Basis and Casis surviving probabilities in the sub-game (ABC) concerning strategies triple N := (A→B; B→A; C→O):

$$P_A^N(ABC) = p_A P_A(CA) + q_A P_A^N(BCA) = p_A P_A(CA) + q_A q_B P_A^N(CAB) =$$
$$= p_A P_A(CA) + q_A q_B P_A^N(ABC). \text{ Solving, we get } P_A^N(ABC) = \frac{p_A P_A(CA)}{1-q_Aq_B}.$$

$$P_B^N(ABC) = q_A P_B^N(BCA) = q_A [p_B P_B(CB) + q_B P_B^N(CAB)] =$$
$$= q_A p_B P_B(CB) + q_A q_B P_B^N(ABC)]. \text{ Solving, we get } P_B^N(ABC) = \frac{q_A p_B P_B(CB)}{1-q_Aq_B}.$$

$$P_C^N(ABC) = p_A P_C(CA) + q_A P_C^N(BCA) = p_A P_C(CA) + q_A [p_B P_C(CB) + q_B P_C^N(CAB)] =$$
$$= p_A P_C(CA) + q_A p_B P_C(CB) + q_A q_B P_C^N(ABC).$$

Solving, we get $P_C^N(ABC) = \frac{p_A P_C(CA) + q_A p_B P_C(CB)}{1-q_Aq_B}$.

As a check on our formulas, note that $P_A^N(ABC) + P_B^N(ABC) + P_C^N(ABC) =$

$$= \frac{p_A P_A(CA)}{1-q_Aq_B} + \frac{q_A p_B P_B(CB)}{1-q_Aq_B} + \frac{p_A P_C(CA) + q_A p_B P_C(CB)}{1-q_Aq_B} = 1$$

Remark. The symbols $P_A^N(ABC)$, $P_B^N(ABC)$, $P_C^N(ABC)$ reflect the sub-game (ABC) and strategies triple N = (A→B; B→A; C→O) choosen as well.

What strategy does Ansis choose? Our intuition predict us that Ansis will try to eliminate the most dangerous enemy, namely, the enemy with higher hit probability. That is Basis. In Basis turn purposeful is eliminate Ansis. For the weakest shooter Casis purposeful is shot in the air in order the strongest enemies shot each other.

Let us stress that just told mathematically is incorrect. That is typically for game theory: correct are solely statements inside the frame of mathematical concepts, any other debate as usually is defective and leads to the wrong conclusions. By my opinion directly that creates objective difficulties for the wide applications of the game theory in the research of social-economical processes.

The next lemma has universal character.

Lemma. Conditional Nash equilibrium of the sub-game.

Let us assume, that $0 < p_C < p_B < p_A \leq 1$; suppose that we have freely chosen some sub-game. Let us assume that Casis as strategy leader decides shot in the air. Than situation associated with strategy triple N := (A→B; B→A; C→O) is conditional Nash equilibrium.

Let us mark the idea of proof.

Let us suppose that sub-game is determined by sequence (ABC) and Casis chooses strategy C→O. We will prove that situation N = (A→B; B→A; C→O) is the conditional Nash equilibrium of the sub-game with corresponding payoffs $P_A^N(ABC)$, $P_B^N(ABC)$, $P_C^N(ABC)$.

The proof for each sub-game consists from two steps.

First, we make certain that in the situations S := (A→C; B→A; C→O), T := (A→O; B→A; C→O) probability for Ansis to survive decreases to compare with probability $P_A^N(ABC)$.

Let us calculate:

$$P_A^S(ABC) = p_A P_A(BA) + q_A P_A^S(BCA) = p_A P_A(BA) + q_A q_B P_A^S(CAB) =$$

$$= p_A P_A(BA) + q_A q_B P_A^S(ABC), \text{ Solving, we get } P_A^S(ABC) = \frac{p_A P_A(BA)}{1 - q_A q_B}.$$

Comparing probability $P_A^S(ABC)$ with probability $P_A^N(ABC)$ getting before, and taking in acount that $P_A(BA) < P_A(CA)$, we get $P_A^S(ABC) < P_A^N(ABC)$.

Comparing probability $P_A^T(ABC)$ with probability $P_A^N(ABC)$ is easier:

from $P_A^T(ABC) = P_A^T(BCA) = q_B P_A^T(CAB) = q_B P_A^T(ABC)$ follows that $P_A^T(ABC) = 0$.

In the second step we make certain that in the situations U := (A→B; B→C; C→O), V := (A→B; B→O; C→O) probability for Basis to survive decreases to compare with probability $P_B^N(ABC)$.

Let us calculate:

$$P_B^U(ABC) = q_A P_B^U(BCA) = q_A p_B P_B(AB) + q_A q_B P_B^U(CAB) =$$

$$= q_A p_B P_B(AB) + q_A q_B P_B^U(ABC). \text{ Solving, we get } P_B^U(ABC) = \frac{q_A p_B P_B(AB)}{1 - q_A q_B}.$$

Comparing probability $P_B^U(ABC)$ with probability $P_B^N(ABC)$ getting before, and taking in acount that $P_B(AB) < P_B(CB)$, we get $P_B^U(ABC) < P_B^N(ABC)$.

We have also $P_B^V(ABC) = q_A P_B^V(BCA) = q_A P_B^V(ABC)$, from what follow $P_B^V(ABC) = 0$.

It may be prooved analogically, that situation associated with strategy triple N = (A→B; B→A; C→O) is conditional Nash equilibrium in other five sub-games.

Consequence. The conditional Nash equilibrium in Martin Shubik game. Let us asume that $0 < p_C < p_B < p_A \leq 1$. Let Casis is strategy leader and chooses strategy C→O. Then situation associated with strategy triple N = (A→B; B→A; C→O) is conditional Nash equilibrium in Martin Shubik game.

Although it is not assert in explicit form the content of theme 1.32 "*When Weakness Is Strength*" in Gintis (2009) creates impression, that triple of strategies N = (A→B; B→A; C→O) determines Nash equilibrium in the each sub-game and in the Martin Shubik game. In the example examined in details this statement is true. Multishaped numerical experiments also signalized that this statement could be true. However the aspirations to prove this statement mathematically leads to the conclusion that for some triple of probabilities p_A, p_B, p_C situation associated with triple of strategies N = (A→B; B→A; C→O) is not the Nash equilibrium.

Let us examine, for instance, the sub-game (ABC). Taking in account the proof of lemma it would be proved that strategy triple N = (A→B; B→A; C→O) determines the Nash equilibrium in the sub-game (ABC), if we were be able to prove that in the situations W := (A→B; B→A; C→A), Y := (A→B; B→A; C→B) the payoff of Casis decreases to compare with P_C^N (ABC).

It is possible to get the following expression:

$$P_C^W(ABC) = p_A P_C(CA) + q_A P_C^W(BCA) = p_A P_C(CA) + q_A [p_B P_C(CB) + q_B P_C^W(CAB)] =$$
$$= p_A P_C(CA) + q_A p_B P_C(CB) + q_A q_B [p_C P_C(BC) + q_C P_C^W(ABC)].$$

Solving, we get $P_C^W(ABC) = \dfrac{p_A P_C(CA) + q_A p_B P_C(CB) + q_A q_B p_C P_C(BC)}{1 - q_A q_B q_C}$.

Before we have got the expression $P_C^N(ABC) = \dfrac{p_A P_C(CA) + q_A p_B P_C(CB)}{1 - q_A q_B}$.

We are interested in investigation of the the set $\{(p_A, p_B, p_C)\}$ of inequality $P_C^W(ABC) < P_C^N(ABC)$ solutions and in the set $\{(p_A, p_B, p_C)\}$ of inequality $P_C^Y(ABC) < P_C^N(ABC)$ solutions. The problem arises: how to haracterize the sets of solution of inequalities $P_C^W(ABC) < P_C^N(ABC)$, $P_C^Y(ABC) < P_C^N(ABC)$.
We are interested also in investigation of the the sets of analogical inequality solutions for the other five sub-games in order to discover conditions what guarantee that strategy triple N = (A→B; B→A; C→O) determines the Nash equilibrium.

I could not receive explicit algebraic description for these solution sets. However I have received empirical descriptions of the inequality $P_C^W(ABC) < P_C^N(ABC)$, $P_C^Y(ABC) < P_C^N(ABC)$ solution sets, displayed in the table 1 and in the figure 1. In the table 1 information about minimal value of probability p_C (depending of values p_A, p_B) is given; with this probability or bigger probability in the situation, determined by strategy triple N = (A→B; B→A; C→O), Casis does not refuse the strategy C→O. If, in addition $p_C < min\{p_A, p_B\}$, then strategy triple N = (A→B; B→A; C→O) determines the Nash equilibrium of the Martin Shubik game.

In the figure 1 dates of the table 1 are depicted graphically. For each value of probability p_A, namely, $p_A = 0{,}1$; $p_A = 0{,}2$; $p_A = 0{,}3$; $p_A = 0{,}4$; $p_A = 0{,}5$; $p_A = 0{,}6$; $p_A = 0{,}7$; $p_A = 0{,}8$; $p_A = 0{,}9$, $p_A = 1$ corresponds curve, what show connection between probabilities p_B, p_C, holding value of p_A constant. Higher the probability p_A, higher is dislocated corresponding curve.
The figure 1 could be considered as empirical solution of the Martin Shubik game.

Table 1. *Information about Nash equilibrium conditions in Martin Shubik game.*

p_A	p_B	***min*** p_C	p_A	p_B	***min*** p_C
0,1	0,02	0,7754	0,6	0,5	0,0431

0,1	0,04	0,5041	0,6	0,6	0,0409
0,1	0,06	0,2501	0,7	0,05	0,9242
0,1	0,08	0,0561	0,7	0,1	0,8310
0,1	0,085	0,0326	0,7	0,2	0,5894
0,1	0,09	0,0227	0,7	0,3	0,2653
0,2	0,02	0,8954	0,7	0,4	0,0721
0,2	0,05	0,6973	0,7	0,5	0,0512
0,2	0,10	0,3309	0,7	0,6	0,0462
0,2	0,12	0,1925	0,7	0,7	0,0448
0,2	0,15	0,0483	0,8	0,05	0,9345
0,2	0,18	0,0257	0,8	0,1	0,8549
0,2	0,19	0,0237	0,8	0,2	0,6506
0,3	0,02	0,9325	0,8	0,3	0,3698
0,3	0,05	0,8073	0,8	0,4	0,1059
0,3	0,10	0,5596	0,8	0,5	0,0617
0,3	0,15	0,2874	0,8	0,6	0,0524
0,3	0,20	0,0673	0,8	0,7	0,0493
0,3	0,23	0,0385	0,8	0,8	0,0485
0,3	0,25	0,0328	0,9	0,05	0,9424
0,3	0,28	0,0289	0,9	0,1	0,8732
0,4	0,02	0,9503	0,9	0,2	0,6977
0,4	0,05	0,8600	0,9	0,3	0,4560
0,4	0,10	0,6804	0,9	0,4	0,1666
0,4	0,15	0,4695	0,9	0,5	0,0762
0,4	0,20	0,2407	0,9	0,6	0,0602
0,4	0,25	0,0714	0,9	0,7	0,0546
0,4	0,30	0,0413	0,9	0,8	0,0525
0,4	0,35	0,0347	0,9	0,9	0,0519
0,5	0,02	0,9608	1	0,05	0,9488
0,5	0,05	0,8905	1	0,1	0,8876
0,5	0,1	0,7520	1	0,2	0,7350
0,5	0,2	0,3959	1	0,3	0,5265
0,5	0,3	0,0714	1	0,4	0,2516
0,5	0,4	0,0408	1	0,5	0,0973
0,6	0,05	0,9103	1	0,6	0,0699
0,6	0,1	0,7985	1	0,7	0,0610
0,6	0,2	0,5078	1	0,8	0,0570
0,6	0,3	0,1501	1	0,9	0,0553
0,6	0,4	0,0530	1	1	0,0548

The explanation of the table 1 and figure 1.

For instance, we can read in table 1 and observe visually in figure 1: if $p_A = 0{,}8$; $p_B = 0{,}4$, then for each probability p_C bigger then 0,1059 Casis shots in the air. Let us remember, that in Martin Shubik game Ansis shots Basis and Basis shots Ansis because of $p_C < p_B$, $p_C < p_A$, namely, Casis is the weakest. Therefore, if probability p_C satisfates inequality $0{,}1059 < p_C < min\{p_A, p_B\} = 0{,}4$, then the strategy triple N = (A→B; B→A; C→O) determines the Martin Shubik games' Nash equilibrium. If $p_C < 0{,}1059$, then strategy triple N = (A→B; B→A; C→O) does not determine the Nash equilibrium because of Casis shots Ansis.

If $p_C > 0{,}4$, then Casis shots in the air if conditions A→B; B→A fulfil. But Casis is now stronger than Basis. What reason in this case is for Ansis to shot Basis?

*Another exam*ple. We can read in table 1 and observe visually in figure 1: if $p_A = 0{,}5$; $p_B = 0{,}2$, then for each probability p_C bigger then 0,3959 Casis shots in the air. However with such probability the conditions of Martin Shubik game does not fulfil. If probabilities follow in order $p_B < p_C < p_A$, then in Martin Shubik game Ansis shots Casis not Basis, because of Basis is weakest. But just $p_C < 0{,}2$ immediately Casis shots

Ansis. Obviously, in the case $p_A = 0,5$; $p_B = 0,2$ such probability $p_C < min\{p_A, p_B\}$ that strategy triple (A→B; B→A; C→O) determines Martin Shubik games' Nash equilibrium does not exist.

In figure 1 especially is depicted the point $(p_B, p_C) = (0,8; 0,5)$ in order to illustrate the very special case $(p_A, p_B, p_C) = (1; 0,8; 0,5)$ examined by Herbert Gintis (2009). Let us mark, that this very special case does not expose the complication of surviving game. Our calculations and figure 1 shows: still the probability p_C satisfies inequalities $0,0570 < p_C < 0,8$ strategy triple (A→B; B→A; C→O) determines Nash equilibrium, namely, Casis shots in the air. In the same time if, for instance, $(p_A, p_B, p_C) = (1; 0,8; 0,05)$ then Casis shots Ansis not in the air.

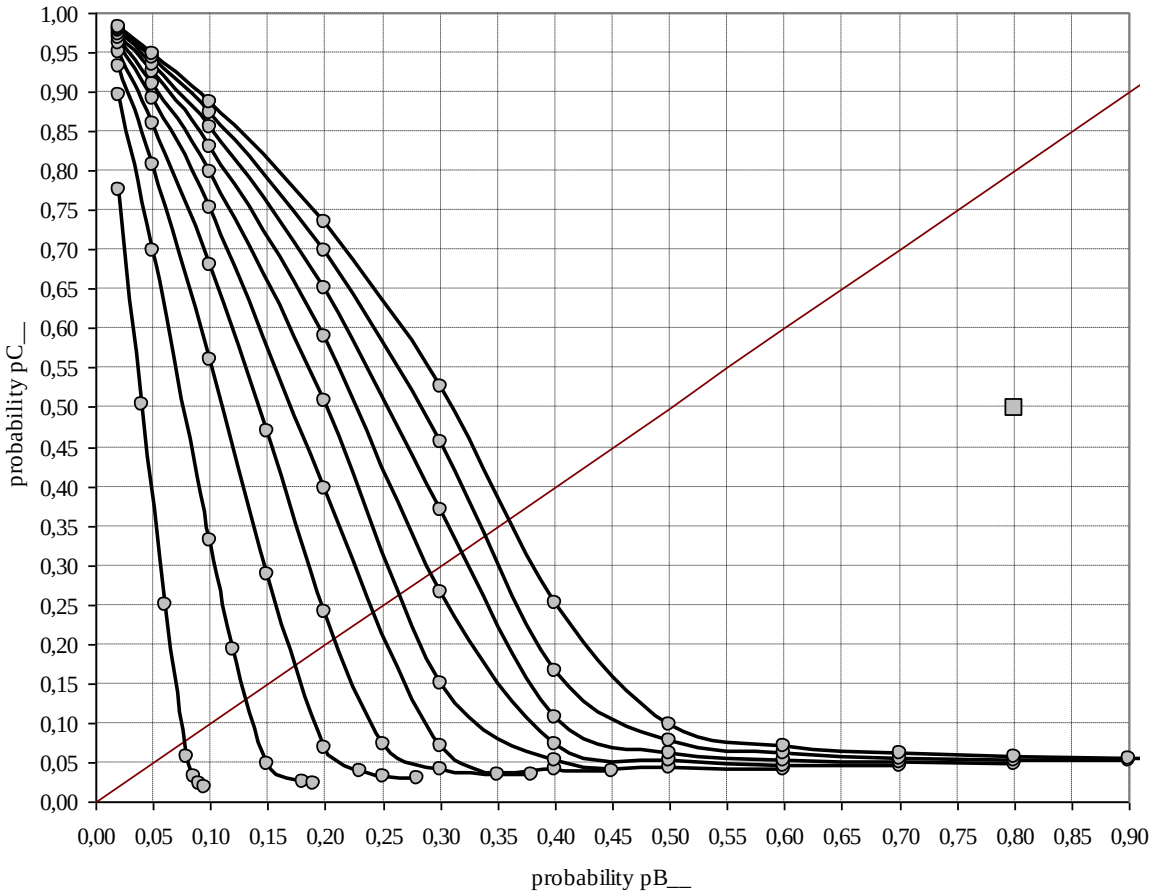

Figure 1. *Graphical solution of the Martin Shubik game.*

3. Some Conclusions Empirically Proven

3.1. Example "When Weakness Is Strength".

Let us theoretical examination of matter done before illustrate now with numerical example. Let us assume, that $p_A = 0,9$; $p_B = 0,6$; $p_C = 0,4$. Note, that probabilities satisfies requirements of table 1: $p_C = 0,4 > 0,0602$.

The surviving probabilities of two agents fighting game are showed in table 2. In table 3 the Nash situations of the sub-games and situations beside to the Nash equilibrium are exposed. In the table 4 the Nash equilibrium of Martin Shubik game is exposed.

Table 2. *Surviving probabilities in the two agents game if $p_A = 0,9$; $p_B = 0,6$; $p_C = 0,4$.*

P_A(AB)	0,93750	P_A(AC)	0,95745
P_A(BA)	0,37500	P_A(CA)	0,57447
P_B(AB)	0,06250	P_B(BC)	0,78947
P_B(BA)	0,62500	P_B(CB)	0,47368

P_C(AC)	0,04255	P_C(BC)	0,21053
P_C(CA)	0,42553	P_C(CB)	0,52632

Table 3. *The Nash equilibriums of the sub-games if $p_A = 0,9$; $p_B = 0,6$; $p_C = 0,4$.*

Sub-game (ABC)	**A**	**B**	**C**	**P_A(ABC)**	**P_B(ABC)**	**P_C(ABC)**
The Nash equilibrium	shot B	shot A	shot air	0,53856	0,02961	0,43183
	shot C	shot A	shot air	0,35156		
	shot B	shot C	shot air		0,00391	
	shot B	shot A	shot A			0,42820
Sub-game (ACB)	A	B	C	P_A(ACB)	P_B(ACB)	P_C(ACB)
The Nash equilibrium	shot B	shot A	shot air	0,53856	0,02961	0,43183
	shot C	shot A	shot air	0,351563		
	shot B	shot C	shot air		0,00391	
	shot B	shot A	shot A			0,42044
Sub-game (BAC)	A	B	C	P_A(BAC)	P_B(BAC)	P_C(BAC)
The Nash equilibrium	shot B	shot A	shot air	0,21543	0,29605	0,48852
	shot C	shot A	shot air	0,14063		
	shot B	shot C	shot air		0,03906	
	shot B	shot A	shot A			0,48396
Sub-game (BCA)	A	B	C	P_A(BCA)	P_B(BCA)	P_C(BCA)
The Nash equilibrium	shot B	shot A	shot air	0,21543	0,29605	0,48852
	shot C	shot A	shot air	0,14063		
	shot B	shot C	shot air		0,03906	
	shot B	shot A	shot A			0,45224
Sub-game (CAB)	A	B	C	P_A(CAB)	P_B(CAB)	P_C(CAB)
The Nash equilibrium	shot B	shot A	shot air	0,53856	0,02961	0,43183
	shot C	shot A	shot air	0,35156		
	shot B	shot C	shot air		0,00391	
	shot B	shot A	shot A			0,34113
Sub-game (CBA)	A	B	C	P_A(CBA)	P_B(CBA)	P_C(CBA)
The Nash equilibrium	shot B	shot A	shot air	0,21543	0,29605	0,48852
	shot C	shot A	shot air	0,14063		
	shot B	shot C	shot air		0,03906	
	shot B	shot A	shot A			0,37459

Table 4. *The Nash equilibrium of the Martin Shubik game if $p_A = 0,9$; $p_B = 0,6$; $p_C = 0,4$.*

The Martin Shubik game	A	B	C	P_A	P_B	P_C
The Nash equilibrium	shot B	shot A	shot air	0,37699	0,16283	0,46018
	shot C	shot A	shot air	0,24609		
	shot B	shot C	shot air		0,02148	
	shot B	shot A	shot A			0,41676

The examined example allows us to formulate following conclusions.

1. Strategy triple (A→B, B→A, C→O) in all six sub-games and, of course, also in Martin Shubik game leads to the Nash equilibrium.

2. In the Martin Shubik game in the Nash equilibrium situation the strategy "shot in the air" insures to Casis the highest surviving probability comparing other two agents. The weakest shooter has the biggest probability to survive! Exactly this result as universal conclusion Gintis (2009) sprightly discuss under the title "When Weakness Is Strength". By opinion of Herbert Gintis this result contradicts to the fundamental thesis of Charles Darvin "*survival of the fittest*".

Below in the example 2.2 "When Weakness Is Not Strength" the case when conclusion of Herbert Gintis does not hold is exposed.

3. Let us observe, that in some sub-games, for instance, in sub-game (ACB) the surviving probability of Casis is lower then surviving probability of Ansis. However if Ansis shots Basis and Basis shots Ansis, then in the each sub-game Casis shots in the air.

3.2. Example "When Weakness Is Not Strength".

Suppose $p_A = 0,9$; $p_B = 0,7$; $p_C = 0,2$.

Note, that probabilities satisfies requirements of table 1: $p_C = 0,2 > 0,0546$.

The surviving probabilities of two agents fighting game are showed in table 5. In table 6 the Nash situations of the sub-games and situations beside to the Nash equilibrium are exposed. In the table 7 the Nash equilibrium of Martin Shubik game is exposed.

Table 5. *Surviving probabilities in the two agents game if $p_A = 0,9$; $p_B = 0,7$; $p_C = 0,2$.*

$P_A(AB)$	0,92784	$P_A(AC)$	0,97826
$P_A(BA)$	0,27835	$P_A(CA)$	0,78261
$P_B(AB)$	0,07216	$P_B(BC)$	0,92105
$P_B(BA)$	0,72165	$P_B(CB)$	0,73684
$P_C(AC)$	0,02174	$P_C(BC)$	0,07895
$P_C(CA)$	0,21739	$P_C(CB)$	0,26316

Table 6. *The Nash equilibriums of the sub-games if $p_A = 0,9$; $p_B = 0,7$; $p_C = 0,2$.*

Sub-game (ABC)	A	B	C	$P_A(ABC)$	$P_B(ABC)$	$P_C(ABC)$
The Nash equilibrium	shot B	shot A	shot air	0,72613	0,05317	0,22069
	shot C	shot A	shot air	0,25826		
	shot B	shot C	shot air		0,00521	
	shot B	shot A	shot A			0,21982
Sub-game (ACB)	A	B	C	$P_A(ACB)$	$P_B(ACB)$	$P_C(ACB)$
The Nash equilibrium	shot B	shot A	shot air	0,72613	0,05317	0,22069
	shot C	shot A	shot air	0,258263		
	shot B	shot C	shot air		0,00521	
	shot B	shot A	shot A			0,21718
Sub-game (BAC)	A	B	C	$P_A(BAC)$	$P_B(BAC)$	$P_C(BAC)$
The Nash equilibrium	shot B	shot A	shot air	0,21784	0,53174	0,25042
	shot C	shot A	shot air	0,07748		
	shot B	shot C	shot air		0,05208	
	shot B	shot A	shot A			0,24936
Sub-game (BCA)	A	B	C	$P_A(BCA)$	$P_B(BCA)$	$P_C(BCA)$
The Nash equilibrium	shot B	shot A	shot air	0,21784	0,53174	0,25042
	shot C	shot A	shot air	0,07748		
	shot B	shot C	shot air		0,05208	
	shot B	shot A	shot A			0,24170
Sub-game (CAB)	A	B	C	$P_A(CAB)$	$P_B(CAB)$	$P_C(CAB)$
The Nash equilibrium	shot B	shot A	shot air	0,72613	0,05317	0,22069
	shot C	shot A	shot air	0,25826		
	shot B	shot C	shot air		0,00521	
	shot B	shot A	shot A			0,19165
Sub-game (CBA)	A	B	C	$P_A(CBA)$	$P_B(CBA)$	$P_C(CBA)$
The Nash equilibrium	shot B	shot A	shot air	0,21784	0,53174	0,25042
	shot C	shot A	shot air	0,07748		
	shot B	shot C	shot air		0,05208	
	shot B	shot A	shot A			0,21528

Table 7. *The Nash equilibrium of the Martin Shubik game if $p_A = 0,9$; $p_B = 0,7$; $p_C = 0,2$.*

The Martin Shubik game	A	B	C	P_A	P_B	P_C
The Nash equilibrium	shot B	shot A	shot air	0,47199	0,29246	0,23556
	shot C	shot A	shot air	0,16787		
	shot B	shot C	shot air		0,02864	
	shot B	shot A	shot A			0,22250

The examined example allows us to formulate following conclusions.

1. Strategy triple (A→B, B→A, C→O) in all six sub-games and, of course, also in Martin Shubik game leads to the Nash equilibrium.

2. In the Nash equilibrium situation of the Martin Shubik game surviving probability for Casis is the lowest to compare with two others agents. The weakest shooter has the lowest probability to survive! This result contradicts to the conclusion of Herbert Gintis (2009, p. 23).

3.3. The example for approving the Nash equilibrium condition.

Let us suppose, that $p_A = 0,7$; $p_B = 0,2$; $p_C = 0,1$.

The probabilities satisfies requirements of table 1: $p_C = 0,1 < 0,5894$.

The surviving probabilities of two agents fighting game are showed in table 8. Tables 9 and 10 show, that strategy triple (A→B, B→A, C→O) does not lead to the Nash equilibrium.

Table 8. *Surviving probabilities in the two agents game if $p_A = 0,7$; $p_B = 0,2$; $p_C = 0,1$.*

P_A(AB)	0,92105	P_A(AC)	0,95890
P_A(BA)	0,73684	P_A(CA)	0,86301
P_B(AB)	0,07895	P_B(BC)	0,71429
P_B(BA)	0,26316	P_B(CB)	0,64286
P_C(AC)	0,04110	P_C(BC)	0,28571
P_C(CA)	0,13699	P_C(CB)	0,35714

Table 9. *The situations of sub-games.*

Sub-game (ABC)	**A**	**B**	**C**	**P_A(ABC)**	**P_B(ABC)**	**P_C(ABC)**
It is no Nash equilibrium	shot B	shot A	shot air	0,79488	0,05075	0,15437
	shot C	shot A	shot air	0,67867		
	shot B	shot C	shot air		0,00623	
	shot B	shot A	shot A			0,15839
Sub-game (ACB)	A	B	C	P_A(ACB)	P_B(ACB)	P_C(ACB)
It is no Nash equilibrium	shot B	shot A	shot air	0,79488	0,05075	0,15437
	shot C	shot A	shot air	0,67867		
	shot B	shot C	shot air		0,00623	
	shot B	shot A	shot A			0,15784
Sub-game (BAC)	A	B	C	P_A(BAC)	P_B(BAC)	P_C(BAC)
It is no Nash equilibrium	shot B	shot A	shot air	0,63590	0,16917	0,19492
	shot C	shot A	shot air	0,54294		
	shot B	shot C	shot air		0,02078	
	shot B	shot A	shot A			0,19770
Sub-game (BCA)	A	B	C	P_A(BCA)	P_B(BCA)	P_C(BCA)
It is no Nash equilibrium	shot B	shot A	shot air	0,63590	0,16917	0,19492
	shot C	shot A	shot air	0,54294		
	shot B	shot C	shot air		0,02078	
	shot B	shot A	shot A			0,20833
Sub-game (CAB)	A	B	C	P_A(CAB)	P_B(CAB)	P_C(CAB)
It is no Nash equilibrium	shot B	shot A	shot air	0,79488	0,05075	0,15437
	shot C	shot A	shot air	0,67867		
	shot B	shot C	shot air		0,00623	
	shot B	shot A	shot A			0,17112
Sub-game (CBA)	A	B	C	P_A(CBA)	P_B(CBA)	P_C(CBA)
It is no Nash equilibrium	shot B	shot A	shot air	0,63590	0,16917	0,19492
	shot C	shot A	shot air	0,54294		
	shot B	shot C	shot air		0,02078	
	shot B	shot A	shot A			0,20650

Table 10. *The situations of the Martin Shubik game.*

The Martin Shubik game	A	B	C	P_A	P_B	P_C
It is no Nash equilibrium	shot B	shot A	shot air	0,71539	0,10996	0,17464

	shot C	shot A	shot air	0,61080		
	shot B	shot C	shot air		0,01350	
	shot B	shot A	shot A			0,18331

The examined example confirms characterization of the Nash equilibrium given into table 1 and in the figure 1 and allows to appraise critically the exposition of the theme "When Weakness Is Strength" in Gintis (2009).

4. Survives agent who is not the weakest but pretends to be weakest

As before we suppose that inequality $0 < p_C < p_B < p_A \leq 1$ holds.

Let us examine surviving game in circumstance when special kind of asymmetric information present.

4.1. Ansis is the strongest but pretends to be weakest.

Let us start with case when Ansis pretends to be weakest. Ansis assures Basis and Casis that probabilities satisfy the inequality $p_A < p_C < p_B$ and he − Ansis will shot in the air. In the reality Ansis is going to shot Basis.

If Basis and Casis believe that $p_A < p_C < p_B$ and A→O, then, as it was showed before, the strategies of Basis and Casis are B→C, C→B, which leads to the illusory conditional Nash equilibrium in the Basis and Casis imaginations. In the reality strategy triple (A→B, B→C, C→B) is realized, and that strategy triple guaranteed to Ansis the highest surviving probability.

Let us illustrate the case when Ansis is the strongest but pretends to be weakest with help of some examples.

Example 1. *"When pretence is strenght".*

Let us asume that in reality $p_A = 0,9$; $p_B = 0,6$; $p_C = 0,4$, but Ansis pretends to be weakest and Basis and Casis believe that $p_A < p_C < p_B$, namely, they believe that Ansis is the weakest and will shot in the air. The situations what arise in the six sub-games as result of strategy triple (A→B, B→C, C→B) are exposed in the table 11. Situation what arise in the Martin Shubik game is exposed in the table 12.

Table 11. *The situations of the sub-games if Ansis pretends to be the weakest.*

Sub-game (ABC)	B	C	A	P_B(ABC)	P_C(ABC)	P_A(ABC)
	shot C	shot B	shot B	0,03842	0,10115	0,86043
Sub-game (ACB)	B	C	A	P_B(ACB)	P_C(ACB)	P_A(ACB)
	shot C	shot B	shot B	0,03842	0,15766	0,80392
Sub-game (BAC)	B	C	A	P_B(BAC)	P_C(BAC)	P_A(BAC)
	shot C	shot B	shot B	0,02305	0,11161	0,86533
Sub-game (BCA)	B	C	A	P_B(BCA)	P_C(BCA)	P_A(BCA)
	shot C	shot B	shot B	0,00231	0,25288	0,74482
Sub-game (CAB)	B	C	A	P_B(CAB)	P_C(CAB)	P_A(CAB)
	shot C	shot B	shot B	0,00384	0,39309	0,60306
Sub-game (CBA)	B	C	A	P_B(CBA)	P_C(CBA)	P_A(CBA)
	shot C	shot B	shot B	0,00231	0,39414	0,60355

Table 12. *The situations of the Martin Shubik game if Ansis pretends to be the weakest.*

Martin Shubik game	B	C	A	P_B	P_C	P_A
	shot C	shot B	shot B	0,01806	0,23509	0,74685

Let us compare this example with example "When Weakness Is Strength" in the section 2.1.
We have $p_A = 0,9$; $p_B = 0,6$; $p_C = 0,4$ in both examples. In the tables 2 and 3 are exposed surviving probabilities of all agents in case when all players now real proportion. In the tables 11 and 12 are exposed surviving probabilities of all agents in case when the strongest player pretends to be weakest, but two other agents do not know true. In the first case in the Martin Shubik game $P_A = 0,38$, in the second case $P_A = 0,75$. It is advantageous do not expose ones power! Let us note, that in the second case very low surviving probability has Basis because of not only Ansis shots to him, but also Casis shots Basis.

Example 2. *"When pretence is strenght".*

Let us asume that in reality the predominance of Ansis is smaller: $p_A = 0,7$; $p_B = 0,6$; $p_C = 0,5$. Ansis pretends to be weakest and Basis and Casis believe that Ansis is the weakest and will shot in the air. The situations what arise in the Martin Shubik game as result of strategy triple (A→B, B→C, C→B) are exposed in the table 13. Obviously, the pretence of Ansis helps him to survive.

Table 13. *The situation of the Martin Shubik game if Ansis pretends to be the weakest.*

Martin Shubik game	B	C	A	P_B	P_C	P_A
	shot C	shot B	shot B	0,06746	0,28692	0,64562

4.2. The second strongest player Basis pretends to be weakest.

Now we are going to demonstrate that pretence strategy provides the highest surviving probability also for the second strongest player − Basis.

For instance, let us examine the case, whe in the reality $p_A = 0,7$; $p_B = 0,6$; $p_C = 0,5$. Basis pretends to be weakest and Ansis and Casis believe that $p_A < p_C < p_B$, namely, they believe that Basis is the weakest and will shot in the air. The situations what arise as result of strategy triple (A→C, B→A, C→A) in the six sub-games are exposed in the table 14. The situation what arises in the Martin Shubik game is exposed in the table 15.

Table 14. *The situations of the sub-games if Basis pretends to be the weakest.*

Sub-game (ABC)	A	C	B	P_A(ABC)	P_C(ABC)	P_B(ABC)
	shot C	shot A	shot A	0,23694	0,09973	0,66332
Sub-game (ACB)	A	C	B	P_A(ACB)	P_C(ACB)	P_B(ACB)
	shot C	shot A	shot A	0,23694	0,13564	0,62742
Sub-game (BAC)	A	C	B	P_A(BAC)	P_C(BAC)	P_B(BAC)
	shot C	shot A	shot A	0,11847	0,19282	0,68871
Sub-game (BCA)	A	C	B	P_A(BCA)	P_C(BCA)	P_B(BCA)
	shot C	shot A	shot A	0,04739	0,33245	0,62016
Sub-game (CAB)	A	C	B	P_A(CAB)	P_C(CAB)	P_B(CAB)
	shot C	shot A	shot A	0,09478	0,41489	0,49033
Sub-game (CBA)	A	C	B	P_A(CBA)	P_C(CBA)	P_B(CBA)
	shot C	shot A	shot A	0,04739	0,45213	0,50048

Table 15. *The situations of the Martin Shubik game if Basis pretends to be the weakest.*

Martin Shubik game	A	C	B	P_A	P_C	P_B
	shot C	shot A	shot A	0,13032	0,27128	0,59840

Numerical experiments performed show us that the surviving probability which Basis gets do to pretending is the largest to compare with another two surviving probabilities. In some sub-games surviving probability of pretenders is even larger then pretenders probability to hit.

5. Conclusions

The author is convinced of cardinal role of game theory in the investigation of individuals' strategy interaction what is fundament for understanding absolutely all social and economic processes. In the circumstance of global confidence crisis the investigation of asymmetric information role in individuals' interactions seems for us especially actual. In present paper the surviving game of three agents, what essence is the competition fight, is investigated. It is established that the shape of Nash equilibrium in a specific way depends of surviving probabilities of the agents. In the paper the limitation of some statements of Herbert Gintis empirically is proved. By opinion of author, the most valuable result of this research is the solution of Martin Shubik surviving game exposed in geometrical form. It is recognized that statement of Martin Shubik "survives the weakest" in circumstances of asymmetric information transforms in statement "survives agent who is not the weakest but pretends to be weakest". My recommendation is to include the surviving game as topic in the game theory course taught in academic program of economics.

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Partially Unforeseen Events. Corrections and Correcting Formulae for Forecasts

Alexander HARIN*

Modern University for the Humanities, Russia

A hypothesis of uncertain future was created and first applied in the field of utility and prospect theories. An extension of application of the hypothesis to the field of forecasting is considered in the article. The concept of inevitability of unforeseen events is a part of the hypothesis of uncertain future, namely of its first consequence. Partially unforeseen events and their role in forecasting are analyzed. Possible applications of the hypothesis in the field of forecasting are considered. Generally, preliminary preparations of forecast corrections are shown to be able, under specified conditions, to quicken the revisions of forecasts after partially unforeseen events have occurred. Particularly, correcting formulae for forecasts are proposed, including additive-multiplicative formulae. The hypothesis of uncertain future, its consequences and their possible applications are briefly reviewed.

Keywords: *forecast, uncertainty, risk, utility, Ellsberg paradox*

JEL Classification: *C53; D81*

1. Introduction

The article is devoted to partially unforeseen events, to their influence on forecasting and to correcting formulae for forecasts in the context of a hypothesis of uncertain future. A brief review the hypothesis of uncertain future, foundations, consequences and applications of the hypothesis are presented. The preliminary analysis of unforeseen and partially unforeseen events is introduced, corrections of forecasts and correcting formulae for forecasts are considered.

1.1. Literature Review

The hypothesis of uncertain future and the correcting formulae for forecasts are, in a sense, pioneering works. So, the main literature consists of the works of the author of this article. Nevertheless, there are a number of works of another authors those are concerned with the item.

A deal of works has been devoted to accuracy and errors of forecasts (see, e.g., Chang, 2011; Morlidge, 2013; McAleer et al., 2008).

Influences of unforeseen, unanticipated events are discussed in the book Hendry and Ericsson Eds., (2003), in the chapters of Clements and Hendry (2006), in articles (see, e.g., Bryan, 2009; Dincer and Kandil, 2011; Lu, 2012; Hackworth et al., 2013; Hendry and Mizon, 2014) and especially in working papers (see, e.g.,

* Corresponding Author:
Alexander Harin, Modern University for the Humanities, Moscow, Russia

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Gonzalez-Hermosillo et al., 2003; Goodhart, 2004; Lahiri et al., 2010; Miller and Ni, 2010; Arbatli and Vasishtha, 2012; Ikeda, 2012; Zeev and Khan, 2013).

The prehistory of hypothesis of uncertain future is presented in Heisenberg's (1927) seminal article about uncertainty principle and in subsequent works.

The article develops earlier reports Harin (2007, 2008, 2010) an article Harin (2012) and a recent report Harin (2014).

1.2. Research Premises. An Example of Hiroshima 1945

The importance of unforeseen events and of partially unforeseen events cannot be overestimated. The unforeseen events can crucially and, sometimes, dramatically change situations. The "black swans" are an example of them. This work serves to smooth down consequences of such events.

As an example, let us suppose that in 1930-35, an imaginary estimate of risk was needed with respect to bombing for an underground factory, government bomb-proof shelter, etc. for the year 1945. Suppose, in 1930-35 the ideal forecast was made. The forecast should be based firstly, e.g., on the forecast of the maximal power of an aircraft bomb for 1945. The forecast should be based secondly, e.g., on the maximal weight that bombing aircraft can lift.

To 1945, due to the most optimistic forecasts, a bombing aircraft could lift a bombing weight much less than 20 tons and even less when calculating in trinitrotoluene equivalent. In 1945 Hiroshima was bombed by the 4-tons atomic bomb. But it was equal to 20000 tons in trinitrotoluene equivalent. So, the initial estimate of risk was catastrophically wrong.

The prerequisite of an atomic bomb (the division of uranium) was discovered in 1938. Naturally, in 1930-35 it was an unforeseen event. So, in this case the relative error, caused by the unforeseen event, is more than 1000 (more than 100000%).

If in 1938 the risk estimate was revised, then the plans and/or the realization of construction of such a factory, shelter, etc. were corrected and then their safety was saved.

1.3. Research Methodology

A hypothesis of uncertain future (see, e.g., Harin, 2007) was created initially in the field of utility and prospect theories. The first purpose of the article is to extend applications of the hypothesis to other fields.

In the field of forecasting there are problems like "black swans" and abovementioned "Hiroshima 1945". The second purpose is to analyze and, if it is possible, partially solve particular problems of the forecasting.

The research objectives correspond to both of the interrelated purposes and are:

Objective 1: A draft of a procedure of application of the first consequence of the hypothesis for particular forecasting cases.

Objective 2: Consideration of general properties of possible formulae of application of the first consequence of the hypothesis for particular forecasting cases.

Objective 3: An attempt to outline a particular formula of application of the first consequence of the hypothesis for particular forecasting cases.

The purposes and objectives are new, so, at first stages of their study, the general methods of the research are mainly qualitative.

Due to the purposes, the used data should be independent and well-established and, so, they should be the experimental results obtained by other researchers and well-known facts. The techniques of analysis of data are mainly qualitative.

This article is a theoretical one. It presents an applied research which develops the basic research (see, e.g., Harin, 2007) of the hypothesis of uncertain future. Methods of this applied research can be attributed to mainly qualitative.

In accordance with the methods of the research, the conclusions are qualitative also.

2. A Hypothesis of Uncertain Future

A hypothesis of uncertain future was presented the first time as a principle of uncertain future in a plenary report Harin (2007) for the field of utility and prospect theories. Let us review it very briefly.

2.1. Formulation of the Hypothesis

A general hypothesis of uncertain future states:

"A future event contains an uncertainty."

A special hypothesis of uncertain future (referred to as simply the hypothesis of uncertain future) states:

"The estimated probability of a future event contains an uncertainty."

Or:

"At present, we cannot actually make an absolutely exact estimate of the probability of a future event" (except imaginary cases).

2.2. Consequences of the Hypothesis

The first (in the preceding works, see, e.g., Harin, 2007), it was denoted as the second) consequence of the hypothesis:

"The present probability system of a future situation is incomplete".

Or:

"Unforeseen events are an inalienable property of the (human) reality".

Or, more particular:

Suppose: After a non-zero interval of time from the present, there will be a future situation. One can calculate the present probability system of this situation. If there will be no unforeseen events, then the total probability of the present probability system of the situation can equal one. But future unforeseen events can emerge during this interval of time. These future unforeseen events can reduce the total probability of the present probability system of the situation by a non-zero value.

The first consequence states:

"The probability of the emergence of these future unforeseen events is non-zero."

The second (in the preceding works, see, e.g., Harin, 2007, it was denoted as the first) consequence of the hypothesis:

"The greater the data dispersion (uncertainty), the smaller the probability of a future event near the probability $p \sim 1$,* and the greater can be** the probability of a future event near the probability $p \sim 0$."

Or, more particular:

"At a non-zero data dispersion, the maximal probability of a future* event is less than one by a non-zero value. If the first consequence influence** is less than the dispersion influence, then the minimal probability of a future* event is also more than zero by another non-zero value"

* This consequence may be regarded as the rigorously proved mathematical statement of the existence theorem for non-zero restrictions on probability. It holds for both future and any other event.

** Because of the first consequence, the magnitude of the low probability is decreased, but because of the second consequence, it is increased. So as a result, it can be either increased or decreased or unchanged.

2.3. Foundations of the Hypothesis. Heisenberg Uncertainty Principle

The general hypothesis of uncertain future can be formally supported by the Heisenberg uncertainty principle.

The Heisenberg uncertainty principle is one of the most distinctive aspects of quantum mechanics. It was devised by Werner Heisenberg at the Niels Bohr Institute in Copenhagen and introduced in Heisenberg (1927).

The Heisenberg's uncertainty principle states that one cannot simultaneously measure both impulse and position better than with uncertainty

$$\Delta p \times \Delta x \geq \frac{\hbar}{2} \ ,$$

where:

Δp - impulse uncertainty;

Δx - position uncertainty;

$\hbar$ - Planck's constant divided by 2π.

The Heisenberg's uncertainty principle is true for every physical object involved in a situation, including future situations. So, it supports the general hypothesis of uncertain future.

2.4. Foundations of the Hypothesis. Existence Theorems for Restrictions

Purely mathematical theorems (see, e.g., Harin, 2010, 2012) prove the probability p cannot reach 1 at the condition of a non-zero dispersion of data. The theorems are presented in a form of a sequence of lemmas and theorems:

For a finite non-negative function on an interval $[0, 1]$, an analog of the dispersion D is proved to tend to 0, when the mean M of the function tends to any border of the interval. Hence, if D is not less than

a non-zero value, then the non-zero restrictions exist on M. Namely, M cannot be closer to any border of the interval, e.g., to 1, than by another non-zero value.

As far as the probability estimation corresponds to such a function and a non-zero dispersion of data takes place, then the non-zero restrictions exist on the probability estimation.

As far as the probability is the limit of the probability estimation and a non-zero dispersion of data takes place, then the non-zero restrictions exist on the probability as well.

3. Partially Unforeseen Events and Corrections of Forecasts

3.1. Fully and Partially Unforeseen Events

There is a wealth of sorts of unforeseen events. According to Caporin and McAleer (2010) these events may be represented by univariate and multivariate models depending on the numbers of the events. We may divide them also into two types: fully unforeseen events and partially unforeseen events.

Rigorously speaking, in the presence of fully unforeseen events, we cannot make any reliable forecast. In other words, "If anything can happen, then nothing can be predicted."

Therefore the partially unforeseen events are considered in this article.

3.2. A Demand of Corrections of Forecasts

Suppose the following: There is a future partially unforeseen event which has at least one unknown characteristic, e.g., time, place, volume, intensity, etc. A forecast is prepared and it is based on a selected particular value of the characteristic. The partially unforeseen event takes place with the real value of the characteristic, which differs from the selected one. If there was a possibility to prepare forward corrections of a forecast, those depend on possible particular values of the characteristic, including the real value, then the forecast can be corrected much faster than in the case of preparing a new forecast.

The first consequence of the hypothesis of uncertain future states the possibility of unforeseen events is an inalienable property of the reality. Therefore, a following conclusion may be drawn:

When the preliminary preparing of corrections (those can be used or not used) is, on the average, more effective than the preparing a new forecast after the partially unforeseen event occurs, then this forward preparing of corrections should be done.

This conclusion is evident in the content of a hypothesis of uncertain future. But in the content of the accepted view it is not. So, it should be drawn and recorded.

Let us further consider some possible forms of such corrections of forecasts.

4. General Properties of Approximation Forecasting

4.1. About the Continuity and Differentiability of Approximations

When choosing an adequately detailed time scale, the vast majority of macro-world phenomena are characterized by continuity in time. The discontinuity, the discreteness in time is observed only in quantum phenomena, for example at the birth of elementary particles. Therefore, the description of the phenomena of the macro-world by means of continuous functions is lawful.

Changes in the macro-world phenomena, that is, the acceleration in a particular dimension, requires physical movements, changes of electromagnetic fields, etc. That is, they are also characterized by continuity in time. Therefore, the description of differentiable functions is lawful for the description of the macro-world phenomena.

4.2. On the Validity of the Approximation Forecasting

So far as for the macro-world phenomena the description by differentiable functions is lawful, then the forecasts of these phenomena in the form of approximations is lawful also. In this sense we can say that the future is a continuation of the present. And we can do calculations and estimates for the prediction of future events of macro-world on the basis of data about current status and about rate of change of these phenomena. Naturally, accurate calculations are possible only for sufficiently small time intervals for which this approximation is correct. But an approximation approach is possible for longer intervals of time, as the basis for assessing possible deviations.

Of course, except of the approximation approach, other approaches may be lawful also.

4.3. The Piecewise Smooth Representation for Univariate and Multivariate Models

Let us consider a pure mathematical case of infinitely differentiable analytical forecast functions. Consider a function $F(t) : F(t)$ is infinitely differentiable and analytic in a point t_{Base} of the timeline and on the semi-closed interval $[t_{Base}, t)$.

Let us denote the Taylor series of the forecast function $F(t)$ as $F(t_{Base}, t)$

$$F(t_{Base},t) = F(t_{Base}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Base})}{n!}(t - t_{Base})^n \ ,$$

where $F^{(n)}(t_{Base})$ is the n-th derivative of $F(t)$ in the point t_{Base}.

Suppose there is a rupture of an n-th $: n \geq 1$, derivative of $F(t)$ in the point $t_{Corr,1} \equiv t_1 : t_{Base} < t_1 < t$, $(t_{Corr,0} \equiv t_0 \equiv t_{Base})$, but $F(t)$ is infinitely differentiable and analytic on $(t_{Corr,1}, t)$. Then, for univariate models, we obtain

$$F(t_{Corr,1},t) = F(t_{Corr,1}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Corr,1})}{n!}(t - t_{Corr,1})^n \ ,$$

where $F^{(n)}(t_{Corr,1})$ is the right-side limit of the n-th derivative of $F(t)$ in $t_{Corr,1}$.

By means of the identical transformation we obtain for $F(t)$ at $t_{Corr,1} < t$

$$F(t) = F(t_{Corr,1},t) \equiv F(t_{Corr,1},t) + F(t_{Base},t) - F(t_{Base},t) =$$
$$F(t_{Base},t) + [F(t_{Corr,1},t) - F(t_{Base},t)] \equiv F(t_0,t) + [F(t_1,t) - F(t_0,t)] \ .$$

Denoting the modification of the function $\Delta F(t_{Corr,r-1}, t_{Corr,r}, t) \equiv [F(t_{Corr,r}, t) - F(t_{Corr,r-1}, t)]$, we have

$$F(t) = F(t_{Base},t) + \Delta F(t_{Base}, t_{Corr,1}, t) \ .$$

For $R : R < \infty$, rupture points $t_r : t_{Base} \equiv t_0$, $t_{r-1} < t_r < t : 1 \leq r \leq R$, (see also Castle et al, 2012) we obtain the general piecewise smooth representation for multivariate models

$$F(t) = F(t_0,t) + \sum_{r=1}^{R} \Delta F(t_{r-1}, t_r, t) \ .$$

Suppose a set of sub-functions $\{f_{1r}(t_{r-1}, t_r, t), \ldots, f_{sr}(t_{r-1}, t_r, t), \ldots, f_{Sr}(t_{r-1}, t_r, t)\} \equiv \{f_{sr}(t_{r-1}, t_r, t)\} : S < \infty$, $f_{sr}(t_{r-1}, t_r, t_r) = 0$, of the modification of $\Delta F(t_{r-1}, t_r, t)$ exists such as $\Delta F(t_{r-1}, t_r, t)$ may be represented as $\Delta F(t_{r-1}, t_r, t) = \Delta F(\{f_{sr}(t_{r-1}, t_r, t)\})$ and $\Delta F(\{f_{sr}(t_{r-1}, t_r, t)\})$ is infinitely differentiable with respect to any $f_{sr}(t_{r-1}, t_r, t)$ and analytic on $(\{f_{sr}(t_{r-1}, t_r, t_r)\}, \{f_{sr}(t_r, t_{r+1}, t)\})$. Let us denote a differential operator

$$T = f_{1r}(t_{r-1}, t_r, t) \frac{\partial}{\partial f_{1r}(t_{r-1}, t_r, t)} + \ldots + f_{Sr}(t_{r-1}, t_r, t) \frac{\partial}{\partial f_{Sr}(t_{r-1}, t_r, t)} \ ,$$

where the derivatives are the right-side limits in the point t_r. Then we have

$$\Delta F(t_{r-1}, t_r, t) = \sum_{l=1}^{\infty} \frac{T^l \Delta F(\{f_{pr}(t_{r-1}, t_r, t_r)\})}{l!} \ .$$

Denoting the modification of the function $\Delta F(t_{Corr,r-1}, t_{Corr,r}, t) \equiv [F(t_{Corr,r}, t) - F(t_{Corr,r-1}, t)]$, we have

So, by means of the formal identical transformations we obtain for multivariate models

$$F(t) = F(t_0) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_0)}{n!}(t - t_0)^n + \sum_{r=1}^{R} \sum_{l=1}^{\infty} \frac{T^l \Delta F(\{f_{pr}(t_{r-1}, t_r, t_r)\})}{l!} \ .$$

5. Particular Formulae for Univariate Models

5.1. General Notes

Let us further consider the case of the only rupture point, of the point of correction $t_1 \equiv t_{Corr,1} \equiv t_{Corr}$

$$F(t) = F(t_{Base},t) + \Delta F(t_{Base}, t_{Corr}, t) \ .$$

Probably, the simplest sorts of partially unforeseen events are those having only unforeseen point of time or unforeseen magnitude and being represented by univariate models. Let us consider them further.

Let us suppose a partially unforeseen event with an unforeseen magnitude and/or an unforeseen point of time has taken place at some t_{Corr}.

If we know the unit value $\delta_1 F(t_{r-1}, t_r, t)$ of modification of the function, which corresponds to the unit magnitude of the event influence, and if, at $t > t_{Corr}$, we know the point of time t_{Corr} and we may determine the magnitude M of the influence, then we may denote $\Delta F(t_{r-1}, t_r, t) \equiv M * \delta_1 F(t_{r-1}, t_r, t)$.

If we have known the correction time point t_{Corr}, then we may express the modification of the function $F(t)$ also.

So, for the partially unforeseen events with the unforeseen magnitude and point of time, we may keep the form of the expression unchanged.

5.2. Low-Order Approximations by Sub-Functions

If a rupture of an n-th (where $n \geq 1$) derivative of $F^{(n)}(t)$ in a point t_{Corr} is caused by a foreseen event, then the series of the right-hand limits of the derivatives $F^{(n)}(t_{Corr})$ may be calculated in advance and the forecast may be corrected in advance also. If this rupture is caused by a partially unforeseen event, then sometimes the forecast correction should be performed extremely rapidly.

Let us consider a case of preliminary conditions:

1) The calculation of the right-hand limits of the derivatives $F^{(n)}(t_{Corr})$ (or the explicit calculation of $F(t)$) is very complicated and needs too long time to be admissible.

2) The function $F(t)$ may be represented by means of a finite set of sub-functions $f_s(t) : s = 1, 2, \ldots, S < \infty$ as

$$F(t) = F(\{f_1(t), \ldots, f_S(t)\}) \equiv F(\{f_s(t)\})$$

and the derivatives

$$\frac{\partial^n F(\{f_s(t)\})}{(\partial f_k(t))^n}$$

of ΔF or F may be calculated in advance.

Let us suppose, that after the partially unforeseen event have taken place, the following additional condition is true:

3. The derivatives

$$F(t) = F(\{f_1(t), \ldots, f_S(t)\}) \equiv F(\{f_s(t)\})$$

are happened to do not essentially depend on specific characteristics of this partially unforeseen event and the preliminarily calculated derivatives may be used (or they may be corrected during the admissible time).

Let us suppose, that the first few L terms give sufficient accuracy of approximation. Then

$$\frac{\partial^n F(\{f_s(t)\})}{(\partial f_k(t))^n},$$

where Δ_{Error} is the total error. Note, that Δ_{Error+} errors can essentially differ from Δ_{Error-}. The impact of negative shocks can differ from that of positive shocks (see, e.g., Caporin and McAleer 2011).

For the first order approximation, the general formula may be easily simplified to

$$F(t) \approx F(t_{Base}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Base})}{n!}(t - t_{Base})^n + \sum_{k=1}^{S} \lim_{t_{Corr}+0 \leftarrow \tau} \left(\frac{\partial \Delta F(\{f_s(t_{Base}, t_{Corr}, \tau)\})}{\partial f_k(t_{Base}, t_{Corr}, \tau)}\right) f_k(t_{Base}, t_{Corr}, t_{Corr}) \pm \Delta_{Error}.$$

It may be expressed also as the derivative of a complex function

$$F(t) \approx F(t_{Base}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Base})}{n!}(t - t_{Base})^n + \sum_{k=1}^{S} \lim_{t_{Corr} \leftarrow \tau} \left(\frac{\partial \Delta F(\{f_s(t_{Base}, t_{Corr}, \tau)\})}{\partial f_k(t_{Base}, t_{Corr}, \tau)} \times \frac{\partial f_k(t_{Base}, t_{Corr}, \tau)}{\partial \tau}\right)(t - t_{Corr}) \pm \Delta_{Error}.$$

5.3. Additive-Multiplicative Formulae

Let us suppose that the modification of the forecast function $\Delta F(t_{Base}, t_{Corr}\ t)$ of an object may be exactly or approximately expressed in the form of explicit functions. These functions may be internal (relative to the object), external (relative to the object), periodic, etc, to specialize, specify unified and standardized forecasts to special, specific forecasting objects and situations. Then the modification $\Delta F(t_{Base}, t_{Corr}\ t)$ may be written in a general form as, for example,

$$\Delta F_{Corr}(t_{Base}, t_{Corr}, t) \approx \Delta F_{Corr}(\{f_{Internal,i}(t_{Corr}, t,)\}, \{f_{External,k}(t_{Corr}, t,)\}, \{f_{Periodic,l}(t_{Corr}, t,)\}, \{f_{Special,m}(t_{Corr}, t,)\}, \Delta_{Error}),$$

where and further:

$\{f_{internal,i}\}$ - the set of internal (relative to the object) functions;

$\{f_{external,k}\}$ - the set of external (relative to the object) functions;
$\{f_{periodic,l}\}$ - the set of periodic functions;
$\{f_{special,m}\}$ - the set of specializing, specifying, adapting, concretizing functions to specialize, specify an unified and standardized forecast to special, specific forecasting objects and situations.

The operations of addition and multiplication are, probably, the most common and important ones as in practice so in the pure mathematics (see, e.g., van der Waerden, 1976).

Let us suppose that the partially unforeseen modification of the forecast function $\Delta F(t_{Base}, t_{Corr}\, t)$ may be exactly or approximately expressed by means of additive and multiplicative functions. Here, an additive function implies a function which additively contributes to the forecast. Here, a multiplicative function implies a function which multiplicatively contributes to the forecast.

Let us consider, as a heuristic hypothesis, the following formula

$$F(t_{Base}, t_{Corr}, t) \approx$$
$$[F_{Base}(t_{Base}, t) \times \prod_{m=1}^{M} K_{Multiplicat,m}(t_{Corr}, t) + \sum_{a=1}^{A} \Phi_{Addit,a}(t_{Corr}, t)] \times ,$$
$$[1 \pm \Delta_{Error}]$$

or, omitting the variables and indices,

$$F \approx [F_{Base} \times \prod K_{Multiplicat} + \sum \Phi_{Addit}] \times [1 \pm \Delta_{Error}] ,$$

where and further:

$F(t_{Base}, t_{Corr}, t)$ - the corrected forecast for the moment t;
$F_{Base}(t_{Base}, t)$ - the base forecast for the moment t;
$\prod K_{Multiplicat,m}$ - the product from 1 to M of the multiplicative (absolute) functions (coefficients) for partially unforeseen corrections;
$\sum \Phi_{Addit,a}$ - the sum from 1 to A of the additive (absolute) functions for partially unforeseen corrections;
Δ_{Error} - the total relative error.

For the cases when

$$F_{Base}(t_{Base}, t) \times \prod_{m=1}^{M} K_{Multiplicat,m}(t_{Corr}, t) \neq 0 ,$$

(preferentially for $F \sim F_{base}$) this formula may be written as

$$F(t_{Base}, t_{Corr}, t) \approx F_{Base}(t_{Base}, t) \times$$
$$[\prod_{m=1}^{M} (1 + k_{Multiplicat,i}(t_{Corr}, t))] \times$$
$$[1 + \sum_{a=1}^{A} \varphi_{Addit,a}(t_{Corr}, t)] \times ,$$
$$[1 \pm \Delta_{Error}]$$

or, omitting the variables and indices,

$$F \approx F_{Base} \times [\prod (1 + k_{Multiplicat})] \times [1 + \sum \varphi_{Addit}] \times [1 \pm \Delta_{Error}] ,$$

where and further:

$\prod (1 + k_{Multiplicat,m})$ - the product from 1 to M of the multiplicative (relative) functions (coefficients) for partially unforeseen corrections;
$\sum \varphi_{Addit,a}$ - the sum from 1 to A of the additive (relative) functions (normalized on $F_{Base} \times \prod K_{Multiplicat,m}$) for partially unforeseen (absolute) corrections.

5.4. Transformations

We may easily obtain the transformations between the versions of the formula.

For the multiplicative functions

$$K_{Multiplicat,m} = 1 + k_{Multiplicat,m} .$$

For the additive functions

$$\Phi_{Addit,a} = \varphi_{Addit,a} \times F_{Base} \times [\prod_{m=1}^{M} (1 + k_{Multiplica,m})] \ .$$

5.5. Analytical and Non-Analytical Sub-Functions

It should be noted that a following possibility cannot be excluded:

When the above formulae were first proposed and considered, the analytical sub-functions were assumed. Nevertheless, the final formulae do not impose conditions of analyticity on the sub-functions. Therefore, a possibility of use of non-analytical, general-form sub-functions cannot be excluded. Such sub-functions can be presented, e.g., in graphical or table or computer calculated forms.

Such possibility can essentially expand feasible fields of applications of the proposed formulae.

6. Possible Applications of the Hypothesis

6.1. Applications of the First Consequence of the Hypothesis. Forecasting

Currently, high-quality forecasting is a rather expensive service. The reason is that such forecasting should take into account a large number of characteristics: from the individual characteristics of the customer to the global settings. Therefore, at present, only sufficiently large teams of specialists can develop high-quality forecasts. And high-quality forecasts can be ordered only by the government or sufficiently large and rich firms, corporations.

But in the case of unforeseen events, the forecast can largely lose its value. That is, the period of possible utilization of the forecast can be shortened by such events.

However, forecasting is an integral part of almost any management process. Therefore, forecasting is a service of mass demand, but its high price prevents its widespread dissemination.

The main advantages of the correcting formulae are:

1) a much lower cost of correction for a user, after a partially unforeseen event occurs, in the comparison with a cost of a new forecast;

2) a much shorter time of correction in the comparison with a time of preparing of a new forecast.

Due to these advantages, the correcting formulae can allow, for example:

The correcting formulae for forecasts can essentially increase the possibilities of application of forecasting for medium and small business.

Government orders for the municipal needs can be one of the most promising areas for application of the correcting formulae for forecasts. Here, the combination of a wide market of forecasts, high-quality development of basic forecasts and standardization is possible. It can be especially useful, e.g., in municipal city-planning programs.

In future, the formulae can allow available orders for the needs of the individual forecasts for individuals, that is, it will make available the individual forecasting.

The formulae will allow constructing and assemblage of forecasts from building forecasting blocks, adjustment of the standard forecasts for specific companies and their activities. Such works can be performed by not only large but also small groups of specialists.

6.2. Applications of the First Consequence of the Hypothesis. Ellsberg Paradox

The first consequence of the hypothesis was used initially for explaining of the Ellsberg paradox (see Ellsberg, 1961) and the problems of the incompleteness of systems of preferences and of ambiguity aversion.

The application of the hypothesis to the Ellsberg paradox and to the problems of the incompleteness of systems of preferences and of ambiguity aversion will be considered in an article in near future. The reports (see, e.g., Harin, 2007, Harin 2008) show the first consequence of the hypothesis of uncertain future explain the problems of the incompleteness of systems of preferences, of ambiguity aversion and of the Ellsberg paradox, at least partially.

6.3. Applications of the Second Consequence. Probability Weighting Problems

Kahneman and Thaler (2006) pointed out that the problems of the utility and prospect theories, including Allais (Allais, 1953) and Ellsberg (Ellsberg, 1961) paradoxes, have not been so far adequately solved. An essential part of the abovementioned problems of the utility and prospect theories consists in the problems, those are connected with a probability weighting (see, e.g., Tversky and Wakker, 1995). The probability weighting means that subjects treat the probability p by a function $W(p)$ which is not equal to p.

The second consequence of the hypothesis was applied to the probability weighting problems (in more detail see, e.g., Harin, 2012).

A practical applicability of the second consequence of the hypothesis and its ability to (at least partially) explain the abovementioned problems are supported by the well-established independent experimental data in independent fields (see, e.g., Harin, 2007, 2012, 2014). This is supported by the risk aversion, the risk premium, the underweighting of high and the overweighting of low probabilities, the opposite underweighting/overweighting of gains and losses, the Allais paradox, the "four-fold pattern" paradox, etc.

7. Conclusion

The hypothesis of uncertain future was created in Harin (2007) for the field of utility and prospect theories. The hypothesis states "At present, we cannot actually make an absolutely exact estimate of the probability of a future event".

The first consequence of the hypothesis includes the concept of unforeseen events. In the case of unforeseen events, their negative influence is that the forecast can largely lose its value. That is, the period of possible utilization of the forecast can be shortened by such events. The existence and influence of such events on forecasts is discussed in the literature (see, e.g., Hendry and Ericsson Eds., 2003, Bryan, 2009, Dincer and Kandil, 2011, Lu, 2012, Hackworth et al., 2013).

The first consequence states unforeseen events are an inalienable property of the human reality. So, the general conclusion and possible application of the hypothesis are: the unforeseen events should be explicitly taken into account in forecasting.

The negative influence of the unforeseen events can be lessened by the preliminary risk management in some cases of the partially unforeseen events. For example, if the influence of a partially unforeseen event could be and was preliminary estimated, then this estimate may be used just after such an event has occurred.

When the preliminary preparations of forecast corrections (those can be either used or not used) for a future partially unforeseen event are possible and more effective (on the average) than the preparing a new forecast or its new correction after the event will occur, then the forecast should include these preliminary corrections for this future partially unforeseen event.

Suppose, there is a future partially unforeseen event which has at least one unknown characteristic, e.g., time, place, volume, intensity, etc. If there was a possibility to prepare preliminary corrections of a forecast, those depend on possible particular values of the characteristic, including the real value, then the forecast can be corrected much faster than in the case of preparing a new forecast.

So, a draft of a procedure of application of the first consequence of the hypothesis has been made for particular forecasting cases.

A long-use forecast should contain correcting terms. These terms may have the form of a framework for forecasts – of correcting formulae for forecasts. These correcting formulae for forecasts may be used as a correcting tool for long-use forecasts and as an adapting tool in addition to unified forecasts to apply them to special situations.

The pure mathematical case of infinitely differentiable function $F(t)$ is considered as one of initial stages of consideration of influence of unforeseen events on forecasts. If there is a set of sub-functions $\{f_{pr}(t_{r-1}, t_r, t)\}$ of the modification of $\Delta F(t_{r-1}, t_r, t)$, such as $\Delta F(t_{r-1}, t_r, t) = \Delta F(\{f_{pr}(t_{r-1}, t_r, t)\})$, then, at the particular conditions, the function $F(t)$, which should be corrected after an unforeseen event, may be represented as a general formula in a form of the Taylor series. At the particular conditions, for the first-order approximation, the general formula may be simplified.

In particular, the modification $\Delta F(t_{Base}, t_{Corr}, t)$ of the forecast function may be exactly or approximately expressed in the form of sub-functions. The operations of addition and multiplication are, probably, the most common and important ones as in practice so in the pure mathematics (see, e.g., Waerden van der, 1976). If one supposes that the $\Delta F(t_{Base}, t_{Corr}, t)$ may be exactly or approximately expressed by means of additive and multiplicative sub-functions (those need not to be expressed in an explicit analytical form), then, omitting the variables and indices, the correcting formula may be written as

$$F \approx [F_{Base} \times \prod K_{Multiplica} + \sum \Phi_{Addit}] \times [1 \pm \Delta_{Error}] \ .$$

For the case of nonzero product of the first two terms (preferentially for $F \sim F_{base}$) it may be written, omitting the variables and indices, in the form of another correcting formula

$$F \approx F_{Base} \times [\prod (1 + k_{Multiplica})] \times [1 + \sum \varphi_{Addit}] \times [1 \pm \Delta_{Error}] \ .$$

So, the general properties of possible correcting formulae have been considered. The particular correcting additive-multiplicative formulae have been created. In general, the application of the first consequence of the hypothesis has been qualitatively extended to the new field and tested in it, the particular problems of the forecasting have been analyzed and their possible solutions have been considered.

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Slutsky Equation and Negative Elasticity of Labor Supply: Behavioral Bias or Optimal Consumption-Leisure Choice?

(For the centenary of "Sulla teoria del bilancio del consumatore")

Sergey MALAKHOV*

Pierre-Mendès-France University, Grenoble, France

One of the applications of the prospect theory is the behavioral phenomenon of the negative elasticity of the individual labor supply. This paper argues that the negative elasticity of labor supply can be understood better with the help of the interpretation of the Slutsky equation with regard to the common consumption-leisure choice. The interpretation of the Slutsky equation corresponds to the empirical evidence that leisure is a net complement for an important part of consumption.

Keywords: *Slutsky equation, prospect theory, labor supply, consumption-leisure choice*

JEL Classification: *D11.*

1. Introduction and Literature Review

In July 1915 the Italian *Giornale degli Economisti* published the article *Sulla teoria del bilancio del consumatore* written by Russian statistician and economist Eugen Slutsky. The further fate of the paper is well known. From the present point of view the destiny of this article looks like a real detective story even in the discreet and profound presentation of J.S. Chipman and J.-S. Lenfant: "As it now well known, Slutsky's article is one of the most famous examples of those neglected and ignored works whose originality and importance are recognized only after similar results have been obtained by others." (Chipman and Lenfant, 2002, p.553) Another discovery of Eugen Slutsky was accompanied by an even greater detective story. However, when R.G.D. Allen, J.R. Hicks, and H. Schultz recognized the Eugen Slutsky's priority in the discovery of the substitution and the income effects, R.Frisch appreciated much more moderately the importance of Eugen Slutsky's article on the summation of random causes as the source of cyclical processes (Slutsky, 1927 and 1937), although "later historians have suggested that it was Slutsky's 1927 article that helped Frisch to construct a mathematical model of the trade cycle in which the oscillations were caused by exogenous shocks." (Barnett 2006, p.420). Hence the name *Economometrics*, given by Eugen Slutsky to the new economic science at birth, did not stick and gave way to the name of *Econometrics*.

However, the question of priority is of secondary importance with regard to the outcome of both Eugen Slutsky's discoveries that provided foundations for much of both neo-classical consumer theory and real business cycle theory. These findings have one common feature in its base – Eugen Slutsky's belief that if an economic phenomenon occurs, than no matter how random or irrational it looks, it should have a rigorous mathematical explanation. There is no doubt that Milton Friedman who got a real historical chance to

* Corresponding Author:
Sergey Malakhov, Ph.D., Applied Economics, Pierre-Mendès-France University, Grenoble, France

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participate in the rediscovery of *Sulla teoria del bilancio del consumatore* in H. Schultz's team shared that belief when he described the positive approach to the economic theory. Unfortunately, the occasional reduction of the positive approach to the famous "as if" notation from the famous billiard metaphor of M. Friedman and L.J. Savage when they compared economic agents with billiard players, who made their shots *as if* they knew the complicated mathematical formulas (Friedman 1953), initiates rather excessive psychological generalizations of inconsistencies of economic behavior. Almost all of these generalizations try to challenge, more or less successfully, the formal economic modeling. Some of these generalizations are presented like applications of the prospect theory regarding the positive theory of consumer choice (Thaler, 1980). However, the prospect theory, which is widely used in enlightenments of "anomalies and puzzles" of economic behavior, can well explain the choice of the billiards player between two risky shots but it cannot replace the natural laws underlying the trajectories of the balls. It has been already presented that some of applications of the prospect theory, i.e., "behavioral inconsistencies", like the search for big-ticket items, the endowment effect, and the sunk costs sensitivity, could be explained by the marginal analysis of the consumer search behavior (Malakhov 2014a, 2014b).

There is another application of the prospect theory that challenges the traditional economic analysis. In 1997 Camerer et al. presented the results of the analysis of labor supply of New York City cab drivers (Camerer et al. 1997 [2000]). The authors of the paper discovered the negative elasticity of labor supply of inexperienced drivers that seemed to be inconsistent with the classical labor-leisure trade-off. And the revised version of that paper was presented in famous *Choices, Values, and Frames* of D. Kahneman and A. Tversky as one of the applications of the prospect theory.

The present paper argues that the negative elasticity of labor supply of inexperienced cab drivers can be understood better with the help of the interpretation of the Slutsky equation with regard to the common consumption-leisure choice.

2. Interpretation of the Slutsky Equation

In 1972 the *American Economic Review* published the article of Ph. J. Cook, graduate student of the University of California, with the elegant interpretation of the Slutsky equation (Cook 1972) Later "Microeconomic Theory: basic principles and extensions" provided the illustrative adaptation of that "one-line" proof for students and instructors (Nicholson 1992, pp.148-150). If we slightly change the way of the "one-line" proof, we can get the illustrative interpretation of the Slutsky equation for the consumption-leisure choice.

When we analyze the negative labor supply elasticity it might be better to choose the indirect statement of the problem. We can replace the question "why the inequality $\partial L/\partial w<0$ occurs?" by the question "why the interrelated inequalities $\partial L/\partial P>0$ and hence $\partial H/\partial P<0$ take place for the given wage rate?" If we follow this indirect statement with regard to the consumption-leisure choice we come to the differential $dQ(P,H(P))$ where we can await both income and substitution effects. It is easy to show that this consumption differential is irrelevant to the labor-leisure choice for the given time horizon, or $dQ(P,H(P))= dQ(P,L(P))$. And we get:

$$dQ(P,H(P)) = dQ(P,L(P))$$

$$dQ(P,H(P)) = dP\left(\left.\frac{\partial Q}{\partial P}\right|_{Hconst} + \frac{\partial Q}{\partial H}\frac{\partial H}{\partial P}\bigg|_{U(Q,H)const}\right) = dP\left(\left.\frac{\partial Q}{\partial P}\right|_{Lconst} + \frac{\partial Q}{\partial L}\frac{\partial L}{\partial P}\bigg|_{U(Q,H)const}\right) \quad (1)$$

$$\partial Q/\partial H < 0; \partial H/\partial P > 0; \partial Q/\partial L > 0; \partial L/\partial P < 0$$

where the bottom line represents the set of common theoretic assumptions underlying consumer behavior.

We can compare graphically this interpretation with the Slutsky equation itself (Fig.1):

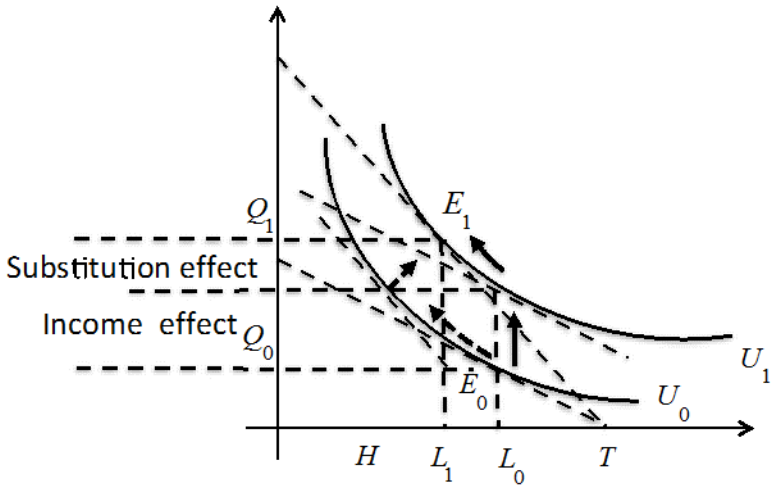

***Figure 1**.Graphical interpretation of the Slutsky equation*

We can follow the prices' fall from E_0 to E_1 along the dotted arrows. However, it is also possible to get the same way along the bold arrows. First, we come to the new utility level for the given allocation of time (L_{const} ; H_{const}). This shift gives us the net income effect for the given income wL. Second, we get the substitution differential dQ where we multiply the change in labor supply by the original marginal rate of substitution of leisure for consumption ($dQ=dL\times\partial Q_0/\partial L_0=dL\times w/P_0=-dL\times\partial Q_0/\partial H_0$). Then we can include the constant wage rate into the substitution differential and get the total derivative $dQ(P,L(P))/dP$:

$$\frac{dQ(P,L(P))}{dP}=\frac{\partial Q}{\partial P}\bigg|_{Lconst}+\frac{\partial Q}{\partial L}\frac{\partial L}{\partial P}\bigg|_{U(Q,H)const}=\frac{\partial Q}{\partial P}\bigg|_{wLconst}+\frac{\partial Q}{\partial wL}\frac{\partial wL}{\partial P}\bigg|_{U(Q,H)const} \quad (2)$$

This is the final result of the interpretation of the Slutsky equation, where the value $\partial Q/\partial P|_{wLconst}$ represents the income effect and the value $\partial Q/\partial P|_{U(Q,H)const}$ represents the substitution effect. However, it gives us only approximated results and it looks not yet illustrative. Nevertheless, its elasticity form can justify the interpretation itself as well as its approximated results:

$$\frac{P}{Q}\frac{dQ(P,L(P))}{dP}=\frac{P}{Q}\frac{\partial Q}{\partial P}\bigg|_{wLconst}+\frac{P}{Q}\frac{\partial Q}{\partial wL}\frac{\partial wL}{\partial P}\bigg|_{U(Q,H)const}\frac{wL}{wL};$$

$$e_{Q,P}=e_{Q,P}\big|_{wLconst}+e_{Q,wL}\,e_{wL,P}\big|_{U(Q,H)const}=-1+1\times e_{wL,P}\big|_{U(Q,H)const} \quad (3)$$

$$e_{Q,P}=-1+e_{wL,P}\big|_{U(Q,H)const}$$

However, even if the total price elasticity of consumption is held negative ($e_{Q,P}<0$), it gives us two different outcomes:

$$\partial H/\partial P\,|_{U(Q,H)const}>0\Rightarrow\partial wL/\partial P\,|_{U(Q,H)const}<0\Rightarrow e_{Q,P}<-1;$$

$$\partial H/\partial P\,|_{U(Q,H)const}<0\Rightarrow\partial wL/\partial P\,|_{U(Q,H)const}>0\Rightarrow e_{Q,P}>-1. \quad (4)$$

We see that for the inelastic demand ($-1<e_{Q,P}<0$) the ***leisure becomes the net complement for consumption***. While this conclusion doesn't correspond to the theoretic properties of the world of two goods, it finds the confirmation in the real world, where the «*empirical evidence indicates that leisure is a net complement for an important part of total consumption*." (Rousslang and Tokarick 1995, p.83). Moreover, the graphical presentation of the prices' fall with regard to stable preferences and the stable north-east-east consumption path (Q/H_{const}) tells us that the net leisure complementarity is really the common case (Fig.2):

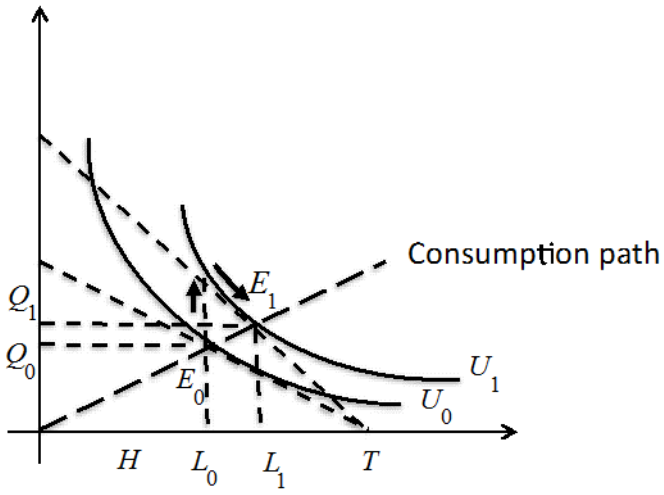

***Figure 2**. Graphical interpretation of the Slutsky equation for the stable inelastic demand*

We can see that here the substitution effect decreases the income effect. This example tells us more about individual labor supply under inelastic demand that the behavioral bias as the application of the prospect theory, used by C. Camerer and his colleagues.

3. Conclusion

The authors of the paper '*Labor Supply of New York City Cab Drivers: one day at a time*' (Part V. *Applications* in *Choices, Values, and Frames*) ask readers in the conclusion to their paper: "*Critics who think our findings of negative elastiticities are an econometric fluke must explain why we did not find negative elasticities for experience drivers.*" The answer could be very simple – because the consumption of experience drivers was elastic while the consumption of inexperienced drivers, i.e., newcomers and beginners with basic needs, was inelastic and it resulted in the negative labor supply elasticity.

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